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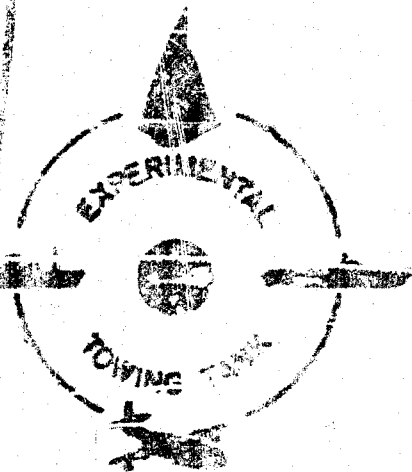
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REPORT NO. 659

May 1957

PITCHING AND HEAVING MOTIONS OF A SHIP
IN REGULAR WAVES

by
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and
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PITCHING AND HEAVING MOTIONS OF A SHIP

IN REGULAR WAVES¹

by

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and

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REPORT 659

22 May 1957

Issue of this advance copy is on the express understanding that no publication, either of the whole or in abstract, will be made until after the paper is made available for discussion in connection with the Annual Meeting of the Society of Naval Architects and Marine Engineers in November 1957

ABSTRACT

As a sequel to the 1955 paper by Korvin-Kroukovsky this study represents an additional step in the gradual development of the theory of ship motions in regular waves which was started by Krilloff in 1896 and which has gained momentum in recent years in the work of Weinblum, St. Denis, Korvin-Kroukovsky and Lewis. Certain errors and omissions of the previous work are corrected and the theory in its present form is verified by comparison of the theoretically computed motions of eight widely different ship models with the motions observed in towing-tank tests over a range of model speeds and at several wave lengths. An excellent correlation is demonstrated in the case of a destroyer model, the shape of which conforms most closely to the assumptions of the theory. Good agreement is found in the cases of several typical commercial ship forms. The agreement is generally satisfactory both in amplitudes of heaving and pitching motion and in the phase relationships between the motions and

¹The material in this paper was prepared at the Experimental Towing Tank, Stevens Institute of Technology as a part of the activity under Office of Naval Research Contract No. Nonr 263-10 (ETT Project FX 1699). Reproduction in whole or in part is permitted for any purpose of the U. S. Government.

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the waves. The theory is found to fail only for a sailing yacht characterized by large slopes of sides at the LWL throughout its length and by a pronounced box overhang.

A discussion is given of the usefulness of the theoretically computed responses to regular waves in predicting ship motions in an irregular sea and a satisfactory agreement is shown for the destroyer model between the statistical averages of heaving and pitching amplitudes of motion obtained in this way and those obtained experimentally.

INTRODUCTION

In a paper written for the November 1955 meeting of the Society of Naval Architects and Marine Engineers, the first author presented a theory of the heaving and pitching motions of a ship in regular head or following seas. That paper added one more chapter to the historic development which was started by Kriloff (1896, 1898)⁴, and which received a strong impetus in recent years from the work of Weinblum and St. Denis (1950), St. Denis (1951) and Korvin-Kroukovsky and Lewis (1955). The particular advances made by the first author in his 1955 paper were consideration of heave-pitch coupling and a theoretical

⁴The Bibliography at the end of the paper is arranged alphabetically by authors' names, with the date of publication given in parenthesis.

calculation of the exciting forces exerted on a ship by waves the results of which were in satisfactory agreement with experimental measurements. The calculation took into account the interaction between ship and wave which is neglected in the heretofore universally applied "Froude-Kriloff hypothesis".

The motions of two ship models computed on the basis of the theory were compared with the motions measured in a towing tank. A reasonably good agreement in the amplitudes of pitching and heaving was demonstrated, but there was a large systematic error in the phase relationships.

In the 1955 paper a "strip" method of analysis was used, not only because of its mathematical simplicity, but also because by this method the distribution of hydrodynamic loads along the hull could be obtained. This distribution combined with the distribution of inertial loads resulting from the ship motions can be directly applied to the analytical calculation of bending moments acting on a ship structure in waves. Since publication of that paper, the second author has been engaged in evaluating ship bending moments on this basis⁵ for comparison with experimental results. It was found in the process that bending moment

⁵Under the auspices of Panel S-3 of the Hull Structure Committee and Panel H-7 of the Hydrodynamics Committee of the Society of Naval Architects and Marine Engineers.

calculations require a much greater precision than calculations of motions, and that the theoretical method which gave reasonably good amplitudes of motion was not yet adequate for the calculation of bending moments.

In the course of re-examination of the original method occasioned by this difficulty, several errors and omissions were discovered and rectified. Most important has been the realization that the velocity-dependent (or damping) terms of the equations of motion consist not only of energy dissipative parts but of dynamic non-dissipative parts as well. The existence of the latter had been demonstrated by Haskind (1946) and Havelock (1955), but not sufficient attention had been paid to their studies, perhaps because of the difficulty in following the mathematics of the first and because of the relatively recent appearance of the second.

Introduction of the necessary corrections has improved the correlation between the calculated and experimental amplitudes of motion and has resulted in a generally good correlation of the phase relationships. Furthermore, confidence in the method has increased with more extended application from the two models studied in 1955 to a total of eight models of widely different types at present. The pitching and heaving motions of these models in regular waves have been experimentally

measured⁶ and analytically calculated, and the results are described in this paper.

In his paper of 1955 the first author emphasized that his objective was to place in the hands of naval architects a working tool to use in the process of developing better ship forms. It is the aim of the present paper to present an improved tool and a broader basis for appraisal of its success in application.

THE PATTERN OF SEAKEEPING RESEARCH

Before proceeding with a detailed analysis of ship motions in regular head seas, it may be advisable to demonstrate how this particular subject fits into the general pattern of research into the seakeeping qualities of ships. The complete picture would require an investigation of ship motions in all six modes — surge, sway, heave, roll, pitch and yaw — in realistic complex seas of any direction. Such a complete investigation has not yet been made, but as a first step the simpler problem of ship motions in head or following seas has been tackled by several writers, beginning with Kriloff (1896). A further simplification has been to assume regular seas, so that side sway, rolling⁷ and

⁶ The authors are indebted to Prof. Edward V. Lewis, Mr. Edward Numata and Mr. John Dalzell for all experimental data.

⁷ Grim (1952) shows that at certain speeds in a following sea rolling instability develops despite all symmetry.

yawing motions are eliminated and only surging, heaving and pitching motions remain. Of these, surging appears to have little effect on heaving and pitching, so that it is sufficient to consider the motions in these two modes. Only in certain aspects of towing-tank testing technique has surging appeared to be important (Vedeler, 1955, Sibul, 1956, and Reiss, 1956).

Formerly the response of a ship to regular waves was thought of as an approximation to the actual behavior at sea. Since the publication of papers by St. Denis and Pierson (1953) and by Fuchs and McCamy (1953), and the subsequent work of Korvin-Kroukovsky and Lewis (1955), Lewis (1955), Lewis and Numata (1957), Cartwright and Rydill (1957) and Cartwright (1957), the point of view has changed. Obtaining the response of a ship to regular waves is no longer considered as an end in itself, but rather as the end result of the hydro-mechanical phase of the broader problem, which is to be followed by a statistical phase in order to obtain the response to realistic complex (or irregular) seas. It should be emphasized that the ship properties such as shape and mass distribution are considered only in the hydro-mechanical phase; the statistical phase is a strictly mathematical treatment of the results obtained in the first. The results of the first phase are required only to be in the form of a plot of amplitudes of pitch and heave per

unit wave height versus frequency of encounter. Various methods of non-dimensional plotting proposed and used from time to time have little usefulness in this connection.

The ship response to regular waves can be measured in a towing tank without any reference to theory. However, experimental data do not bring out readily the significance of the various physical properties of a ship. It is necessary to develop a reasonably comprehensive theory in order to gain an understanding of the effects of these properties on the motions of a ship. Such a theory would also provide a basis for judging the experimental data. There are many pitfalls in towing tank testing of ship models in waves which can be avoided when knowing what to expect on theoretical grounds.

The theoretical approach to the problem of ship responses to regular waves has taken two general directions. One is to undertake the complete problem of a body oscillating in waves at once, i. e. to include initially in the equations of motion the dynamics of the body and the hydrodynamics of a fluid with a free surface. Probably the most complete study of this kind is that of Haskind (1946), which has been made accessible to English-speaking readers by Dr. Wehausen's translation (under the auspices of the H-5 Panel of the Society of Naval Architects and Marine Engineers). These studies have been

valuable in providing guidance to the simpler approaches, but their mathematical difficulty and the need for extensive calculations to bring out the final results prevent their being used directly by naval architects. The other, simpler, approach has been taken by Kriloff (1896, 1898), Weinblum and St. Denis (1950) and St. Denis (1951). Here the differential equations of motion are written on the basis of the dynamics of rigid bodies and various coefficients are independently and individually evaluated by hydrodynamic considerations. They are usually derived by analogy with a fully submerged body and the effects of a free water surface are allowed for by separate considerations. Specialized investigations of virtual masses and damping such as those by Holstein (1936, 1937 a,b), Havelock (1942), Ursell (1949, 1953, 1954), and Grim (1953) are available, and while these studies are by no means uninvolved the results are presented in simple form and can be readily included in the solution of the equations of motion. With this supporting literature the simpler theoretical approach has become more and more realistic.

The present paper follows this simpler approach. Its immediate objective is the calculation of ship responses to regular head or following seas, with the expectation that this material can then be statistically treated to permit prediction of ship behavior in a realistic

irregular sea. In addition it is desired to provide a method that can be readily extended to the calculation of ship bending moments.

THE EQUATIONS OF MOTION

Neglecting the surging, the equations representing the equilibrium of forces and moments acting on a ship moving in regular head or following seas can be written in the form

$$\begin{aligned} m \ddot{z} + c z &= H_f \\ J \ddot{\theta} + C \theta &= H_m \end{aligned} \tag{1}$$

where m is the mass of the ship itself and J is its moment of inertia; z is the heaving displacement and θ the angular displacement in pitch; c and C are the unit restoring force and moment respectively due to changes in the buoyant forces arising from the deviation of a ship from its attitude of normal flotation; H_f is the force and H_m the moment about the center of gravity of a ship due to water pressures generated in the flow by the action of the wave as well as by the ship's own oscillations. These forces and moments are computed in the Appendix, and are found to be expressible by polynomials the terms of which are divided into those corresponding to the ship's oscillation in smooth water and those corresponding to direct wave action on a restrained ship. When the terms of the first kind are

transferred to the L.H. side of Equations (1), the equations take the form

$$\begin{aligned} a \ddot{z} + b \dot{z} + c z + d \ddot{\theta} + e \dot{\theta} + g \theta &= \bar{F} e^{i\omega t} \\ A \ddot{\theta} + B \dot{\theta} + C \theta + D \ddot{z} + E \dot{z} + G z &= \bar{M} e^{i\omega t} \end{aligned} \quad (2)$$

The complete expressions for the coefficients on the L.H. side are given in the Appendix by Equations (42). The terms on the R.H. sides of Equations (2) are known as the exciting force and moment. They are given in complex form in order to facilitate the algebraic work of the solution and it is understood that only the real part of the exponential on the R.H. side of Equations (2) is to be taken. In this form

$$\begin{aligned} \bar{F} &= F_0 e^{i\sigma} \text{ and the real part of } \bar{F} e^{i\omega t} = F_0 \cos(\omega t + \sigma), \\ \bar{M} &= M_0 e^{i\tau} \text{ and the real part of } \bar{M} e^{i\omega t} = M_0 \cos(\omega t + \tau). \end{aligned} \quad (3)$$

Here F_0 and M_0 are respectively the amplitudes of heaving force and pitching moment caused by waves, as computed by Equations (26) and (27) in the Appendix; σ and τ are the phase angles at which the maxima of the force and moment occur (positive angles are leads, negative angles are lags). σ or τ is zero if the maximum occurs at the instant when the wave crest is amidships. The harmonic oscillation of force and moment is given by the factor $\exp(i\omega t)$ in Equations (2), where ω , written for brevity in place of ω_e , is the frequency of wave encounter. The terms in θ in the first and in z in the second of Equations (2) represent

the interaction between heaving and pitching motions. These terms are usually referred to as "cross-coupling terms".

It is assumed that the ship is moving in a uniform sea, and that a steady state of heaving and pitching is established. In such a case, the transient responses have been damped out, and only a particular solution of Equations (2) is required. Since the exciting functions are assumed to be harmonic, the solution is likewise harmonic, i. e. of the form

$$z = \bar{Z} e^{i\omega t} \quad \text{and} \quad \theta = \bar{\Theta} e^{i\omega t} \quad (4)$$

\bar{Z} and $\bar{\Theta}$ are complex amplitudes

$$\bar{Z} = Z_0 e^{i\delta} \quad \text{and} \quad \bar{\Theta} = \Theta_0 e^{i\epsilon}$$

where Z_0 is the absolute value of the heaving amplitude, Θ_0 that of pitching, and $-\delta$ and $-\epsilon$ are the phase lags of ship motion relative to ωt .

The solution of Equations (2) is readily obtained as

$$\bar{Z} = \frac{\bar{M} Q - \bar{F} S}{Q R - \bar{P} S} \quad (5)$$

$$\bar{\Theta} = \frac{\bar{F} R - \bar{M} P}{Q R - \bar{P} S} \quad (6)$$

where the capital letters represent the groupings of the coefficients of Equations (2) as follows:

$$\left. \begin{aligned} P &= -a \omega^2 + i b \omega + c \\ Q &= -d \omega^2 + i e \omega + g \\ R &= -D \omega^2 + i E \omega + G \\ S &= -A \omega^2 + i B \omega + C \end{aligned} \right\} (7)$$

Were the "cross-coupling" coefficients of Equations (2) set equal to zero, i.e. $d = e = g = D = E = G = 0$, the two equations would represent two simple harmonic oscillators in forced motion, and Equations (5) and (6) would take the familiar form of "magnification factor".

THE SHIP MODELS USED FOR COMPUTATIONS AND EXPERIMENTS

The heaving and pitching motions in regular head seas have been calculated from Equations (5), (6) and (7) above for eight ship models of different forms. The properties of these models are given in Table 1 and the body plans in Figs. 1 to 5. These hulls are currently subjects of research at the Experimental Towing Tank of Stevens Institute of Technology for various projects sponsored severally by the David Taylor Model Basin, the Office of Naval Research, and the Society of Naval Architects and Marine Engineers. This collection of models does not therefore represent a series developed for a particular purpose; it includes a wide range of ship forms and displacement-length ratios. These forms may, however, be considered in groups of two or three having certain resemblances and differences.

Model 1445 is the Series 60, 0.60 block coefficient form and Model 1616 is an experimental modification of it by E. V. Lewis (1955) which has the same dimensions and the same afterbody but an extreme V-type forebody (Fig. 1). These two models were investigated by Korvin-Kroukovsky in his 1955 paper but have been submitted to re-analysis by the corrected and improved procedure given in the present paper. The newly-computed values of all the coefficients for wave length λ equal to ship length L , $\lambda/L = 1$, are listed in Tables 2 and 3, which replace Tables 4 and 6 of the earlier paper, and the new curves of B , b , e_2 and E_2 versus frequency and of C and g versus forward speed are shown in Figs. 6 and 7 which replace Figs. 1 to 4 in the earlier paper. The calculated and experimentally measured responses of these models to waves of two lengths, $\lambda/L = 1$ and 1.5, are shown in Fig. 8 which replaces Figs. 5 to 8 in the earlier paper.

Model 1444 is the T-2 Tanker (Fig. 2) which has been the subject of many experiments conducted by E. V. Lewis (1954) in connection with the measurement of bending moments (under the sponsorship of the S-3 Panel of the Society of Naval Architects and Marine Engineers). The calculated and experimentally measured motions for $\lambda/L = 1$ are shown in Fig. 9.

Model 1723 is the destroyer DD692 (Lewis and Dalzell, 1957) of displacement-length ratio 60 (Fig. 3). The results of computations and towing tank tests for four different wave lengths are shown in Fig. 10.

Model 1699A (Fig. 4) is the German fishing trawler "Stralsund", described by Captain W. Möckel (1953) as possessing particularly good seagoing qualities. It is a short stubby hull of displacement-length ratio 204. Model 1699C has the same section forms as the trawler but spaced wider apart and dimensioned to give the low displacement-length ratio of 60 like the destroyer. Fig. 11 shows the calculated and experimentally measured motions of 1699A and 1699C in waves of two lengths, $\lambda/L = 1$ and 1.25. Attention should be called to the different speed scales on the L.H. and R.H. sides of Fig. 11.

Model 1699B (Fig. 5) is the British cruising yacht "Brambling", also reputed to have excellent sea-going characteristics (Fox, 1938). With a displacement-length ratio of 370, it is the stubbiest model of the group investigated. The lengthened counterpart of the yacht, Model 1699D, has the same section forms spaced wider apart and a displacement-length ratio of 60 like the destroyer and the lengthened trawler. Fig. 12 compares the calculated and experimentally measured

motions of 1699B and 1699D in waves of two lengths, $\lambda/L = 1$ and $\lambda/L = 1.25$. (Note difference in speed scales on L.H. and R.H. sides of this figure)

Since such small vessels as trawlers and yachts have succeeded in operating efficiently in an open ocean, it was thought that their forms may have some particularly desirable features worth investigating. E. T. Lewis (1955) and Kuzata and Lewis (1957) have shown the advantages of a low displacement-length ratio so that the high displacement-length ratio of these small ships appears to be a drawback. The reason for the seaworthiness of these hulls must lie, then, in their freeboards and sectional forms.

The destroyer (Model 1723), the lengthened trawler (Model 1699C) and the lengthened yacht (Model 1699D) can be compared for the effect of section form at the same low displacement-length ratio of 60. The effect of section form is also shown on Fig. 3 by comparison of Model 1145, the Series 60 hull, and Model 1616. Model 1616 has the same sectional area curve and the same afterbody as Model 1145 but its forebody is modified to have extreme V sections, more or less like a Maier form.

The effect of greatly reducing the displacement-length ratio,

i. e. of stretching the models, while keeping the same section forms can be seen by comparison of the original and the lengthened versions of the trawler and the yacht on Figs. 11 and 12.

CORRELATION OF CALCULATED AND EXPERIMENTAL RESULTS

Excellent agreement between analytically calculated and experimentally measured results is found on Fig. 10 which depicts the responses of the destroyer (Model 1723) to waves of four lengths, $\lambda/L = 1, 1.25, 1.5$ and 2, which cover the most important operational range. It should be noted that of the eight this model comes closest to being "wall-sided" at the LWL, and in this sense conforms closely to the assumptions made in the theoretical derivations of the Appendix as well as in the supporting literature on damping.

Next in degree of correlation come the Series 60 hull (Model 1445), Fig. 8, and the T-2 tanker (Model 1444), Fig. 9. Here, too, the agreement is excellent between the calculated and experimentally measured pitching motions and phase relationships. However, there is a significant overestimation of the heaving motions at and near synchronism, for both models, and, while the calculated phase lags in heave are close to the measured lags for Model 1445, for Model 1444 there is some discrepancy at speeds higher than 2 ft/sec. Both of these models have U sections

forward, where they deviate little from the assumption of wall-sidedness. Pronounced slopes of the tangents to the section forms at the LWL occur only at the stern where the relative ship-water motions are generally smaller and the effect of section form is less important.

The calculated and observed amplitudes of motion for Model 1616 (lower half of Fig. 8) agree very well at $\lambda/L = 1$, but at $\lambda/L = 1.5$ there is a significant error in the estimate of pitching amplitude. The phase relationships show good correlation in pitch but not in heave. This model, with its extreme V sections at the bow, has, of course, a pronounced slope of the tangents to the sections at the LWL, deviating in this respect from the assumptions of the theory.

In the case of the lengthened trawler Model 1699C (right hand of Fig. 11), there is on the average a very good agreement between the calculated and test results, but the calculations underestimate the pitching amplitudes and do not show the exaggerated narrow peak in heaving at synchronism. The calculated phase relationships agree very well with the test results. As can be seen from Fig. 4, the slopes of the tangents to the bow sections are more moderate for the trawler than for the V-bow Model 1616 but greater than for Model 1445.

The original trawler Model 1699A combines these section characteristics with the high displacement-length ratio of 204. The experimental

and calculated motions agree quite well at $\lambda/L = 1$, but there are large discrepancies in the pitching amplitudes and phase lags at $\lambda/L = 1.25$. The agreement in heave amplitudes and phases at the latter wave length is excellent.

The only models for which the computation method fails completely are the yachts, both the original Model 1699B and the lengthened Model 1699D. These hulls have not only large slopes of the tangents to the sections at the LWL at bow and stern, but also appreciable slope at midship section and, in addition, a large bow overhang and a cutaway forefoot. These deviations from the assumptions of the present theory of ship motions appear sufficient to mutilate computed results.

It was expected that estimates of the motions of the original yacht would be poor not only because of the slopes of the sides at the LWL and the large overhang but also because of its stubbiness. This model was included in the analysis in an attempt to establish the limits of the applicability of the theory. However, the failure in the case of the lengthened yacht with a low displacement-length ratio of 60 is illuminating. It indicates that the section slopes and the end overhang have much greater weight in the calculations than the displacement-length ratio. The relatively small importance of the latter could have been inferred from Vosser's (1956) calculations of three-dimensional

damping forces for three length-beam ratios, as shown in Fig. 17, and from the comparison by Korvin-Kroukovsky (1955a) of his inertial force calculations for a spheroid of length-diameter ratio 5 with Havelock's for an infinitely long spheroid.

To summarize, there is excellent agreement between the calculated and experimentally measured ship motions in the case of a destroyer, and a generally satisfactory agreement in the case of normal ship forms. Only in the case of a yacht is the computation method completely inadequate.

In the discussion above the authors have purposely refrained from mentioning non-linearity. Certainly the slopes of the hull surface at the LWL indicate a strong non-linearity in the restoring forces and the corresponding cross-coupling terms, i. e. in the coefficients c , C , g and G of the equations of motion. However it is not certain whether it is this non-linearity or the inexactness of the virtual mass and damping coefficients which is responsible for the discrepancies between the calculated and the observed motions. Attention should be called to the fact that although the forms for which Grim has derived the damping forces include what appear to be extreme V sections, these sections have a sharp turn of the bilge and are tangent to a vertical at the LWL, whereas the bow sections of a Maier form or of the yacht discussed

above have large slopes at the LWL and so may well have quite different damping properties. Unfortunately neither theoretical nor experimental information on such sections is available. Information available on wide V sections used in speed boats and in seaplane hulls has been derived for very high Froude numbers, at which the force of gravity can be neglected in comparison with acceleration forces, and is not valid for the range of Froude numbers found in ship operation.

THE SIGNIFICANCE OF THE CHARACTERISTIC PROPERTIES OF SHIP FORMS

As a result of all investigations made so far, the effects of certain properties of ship forms have become established. The natural pitching or heaving period of a ship oscillating in smooth water becomes somewhat modified in coupled motion, but nevertheless remains a most important criterion. The shorter the natural period, the higher the frequency of wave encounter at synchronism, and therefore the higher the ship speed for synchronism in waves of any given length. The waves causing large ship motions are equal to or longer than ship length, and E. V. Lewis (1955) has shown that the limiting ship speed in rough weather therefore increases with ship length. This effect is clearly shown on Figs. 11 and 12 by a comparison of the original and lengthened models of the trawler and the yacht⁸.

⁸ Note that the speed scales are different for the original and lengthened hulls.

However, a naval architect will arrive at a certain ship length and a certain displacement-length ratio by many considerations other than seakeeping ability. Whatever the dimensions chosen, it is desirable to see what improvement in seakeeping can be obtained by selecting a suitable ship form. Evidently the most important characteristic here is the damping. The theoretical work of Holstein (1936, 1937a, 1937b), Havelock (1942) and Grim (1953) shows that damping increases rapidly with decrease of the mean draft of a ship section. Yet avoidance of bow emergence and slamming dictates a large actual draft. Both conditions lead to the requirement of a low section coefficient, usually associated with a V form, or at least with a large deadrise of the ship bottom⁹. This is indeed a characteristic of the small ships operating in high seas, the yachts, fishing trawlers and coast-guard cutters.

These principles are well illustrated by the comparison on Fig. 8 of the Series 60 Model 1445 with its modified V-bow version, Model 1616. The larger beams of the bow sections of Model 1616 increase the moment

⁹ The advantage of a low section coefficient in reducing the amplitude of heaving motion in waves is vividly demonstrated in the recent theoretical work of Grim (1957). Of two sections having draft equal to half-beam, the one with section coefficient of 0.555 is shown to oscillate with a magnification factor close to unity through a wide range of frequencies, while the one with section coefficient of 1.015 has a magnification factor of 3 at a sharp synchronous peak.

of inertia of the waterplane and this results in a higher ship speed for synchronism. The smaller mean drafts of the V sections increase the damping forces and thereby markedly decrease the amplitudes of pitching and heaving.

The advantages of the trawler and yacht section forms over those of a conventional ship like the Series 60 lie largely in the increased damping due to smaller section coefficients. There is rather little difference in the behavior of the destroyer (Model 1723) and the lengthened trawler and yacht (Models 1699C and 1699D), which were made to match the displacement-length ratio of the destroyer, because the destroyer also has the desirable low section coefficients. Indeed the very low mean draft of the stern sections of the destroyer gives it somewhat heavier damping and slightly lower amplitudes of motion.

This advantage of the destroyer over the lengthened trawler is offset in part by the fact noted during the model tests (Numata and Lewis, 1957) that the destroyer frequently shipped water more heavily. It appears that concentration of heavy damping at the stern is not as desirable as a more uniform distribution of damping between bow and stern. This conclusion is in agreement with the often-expressed opinion that "double-ended" boats are preferable for good seakeeping qualities, and in fact trawlers, and fishing boats in general, show a distinct trend

in this direction.

The beneficial effect on the behavior of a hull resulting from a low section coefficient with its corollary a large damping coefficient is quite evident qualitatively from the familiar definition of "amplification factor", i. e. the solution of the simple uncoupled equations of motion. It is impossible, however, to judge the effects of the cross-coupling coefficients by inspection of the coupled equations of motion or their solution. Experimentally also these effects are difficult to trace because of the number of different models required and the tediousness of towing-tank testing. So far all analytical efforts have been expended in developing a theory of ship motion and in demonstrating its validity. Until recently the theory was not good enough to warrant the series of calculations necessary to bring out the influence of the cross-coupling coefficients; but now it appears that at least for ship forms not deviating too much from conventional ones the theory is sufficiently reliable to make such computations worthwhile. A computational series of systematic changes of form leading to changes of cross-coupling coefficients could be easily organized and carried out economically on many of the currently available computing machines.

PREDICTION OF MOTIONS IN IRREGULAR SEAS
FROM THE CALCULATED RESPONSES

As stated earlier, the ship responses to regular waves do not indicate directly the behavior in irregular waves. However, a statistical treatment of these responses in combination with the spectrum of sea waves can give results which are significant for the ship motion in the realistic sea. The technique of this process was first described by St. Denis and Pierson (1953) and it was applied to model tests by E. V. Lewis (1955). A simple exposition was presented by Korvin-Kroukovsky (1956, 1957). In this process the square of the amplitude of ship motion per unit wave height, for a given wave length at a particular speed (i.e. for a given frequency of encounter), is multiplied by the ordinate of the sea spectrum (corrected to that ship speed) at the same frequency of encounter. These products plotted against frequencies of encounter (resulting from a range of wave lengths at a particular speed) form the spectrum of the ship motions at that speed. Since the ship motions in an irregular sea are also irregular, varying continuously in amplitude and period, they can only be described statistically, as for instance by mean amplitude, mean of the 1/3 highest amplitudes, or mean of the 1/10 highest amplitudes. The last figure is often taken as the probable near-maximum. These means were derived by Longuet-Higgins (1952) as multiples of the square root of the area E under the response spectrum

curve: the mean amplitude is equal to $.88 \sqrt{E}$, the mean of the 1/3 highest is equal to $1.41 \sqrt{E}$, and the mean of the 1/10 highest is equal to $1.8 \sqrt{E}$. These factors are applicable to the typical spectra of ocean waves as well as to the ship motions in them.

As an illustration of the above, pitch and heave spectra were predicted for the destroyer model No. 1723 from the calculated responses to regular waves of eight lengths at two ship speeds. The squares of the computed responses were divided by the square of the wave height and the quotients were multiplied by the ordinates of the irregular wave spectrum derived from analysis of the waves in the towing-tank tests, thus yielding motion spectra. These spectra were compared with the spectra obtained experimentally by Prof. E. V. Lewis and his associates at the Experimental Towing Tank of Stevens Institute of Technology (Lewis and Dalzell, 1957) who used two different methods: first, the spectra were obtained directly by statistical analysis of irregular model motions, and second, the spectra were computed from experimentally measured responses to a series of regular waves. A comparison of the various spectra is given in Fig. 13, where the solid lines represent the spectra resulting from the statistical analysis of the experimental records of ship motions in irregular waves, the dash lines the spectra computed from the experimental responses to regular waves, and the

crosses the spectra predicted by the methods of the present paper.

It should be emphasized that the shape of the spectrum curve has no significance for the amplitude of ship motion. Components of all frequencies exist simultaneously and only the area of the spectrum, E , is significant. Means of the 10% highest amplitudes, computed from the areas under the spectra, are given in Table 4. The amplitudes of heaving motion obtained by the three different methods are in excellent agreement. The pitch amplitudes computed from responses to regular waves are slightly smaller than those resulting directly from a statistical analysis of the irregular model motions. On the average there seems to be no important difference between the amplitudes computed from the experimental and those computed from the theoretical responses to regular waves (second and third lines of Table 4); at zero speed the theoretically computed values are nearer the statistically derived ones, at 2.53 ft./sec. the values based on experiments in regular waves are closer by about 5%. The deviations are well within the accuracy of the calculation procedure. Since there is always a rather large statistical uncertainty in evaluating sea spectra, further improving the accuracy of the calculation of responses to regular waves would have little meaning. It is only important to look for ways to correct consistent discrepancies or a trend contrary to reality.

The discussion so far has been limited to amplitudes of motion. These are readily obtainable by both experimental and theoretical methods in use at present but they are not important just by themselves. Other phenomena may be more decisive in qualifying the seakindliness of ships. The accelerations, which for a given frequency are proportional to the amplitudes, are more important for passenger and transport ships since there is a direct connection between accelerations and sea-sickness (Geller, 1940, and Shaw, 1954). On fishing trawlers the accelerations impose hardships on the crew at work (Möckel, 1953). On modern cargo ships, with crew accommodations not far from amidships, the accelerations in pitch and heave are of minor importance except for the structure supporting the cargo in the lower decks in No. 1 hold. The critical conditions limiting the operation of these ships appear to be shipping of water and slamming, the latter mostly in the light condition. These conditions probably also limit the operations of such naval ships as destroyers.

The statistical treatment of the accelerations is similar to that of the amplitudes of the motions. The vertical acceleration at the bow, for example, can be derived analytically from the computed motions by the formula $a_0 = \omega^2 s_0$, where a_0 is the amplitude of vertical acceleration, s_0 is the amplitude of vertical displacement obtained by

combining the heaving and pitching motions in the correct phase, and ω is the frequency of encounter. Acceleration spectra can be obtained and from these the mean amplitudes, the mean of the 1/10 highest amplitudes, etc., in irregular waves.

E. V. Lewis (1955) has shown how, by superposition of the path of the ship's bow on the regular wave profile, the bow submergence or emergence can be evaluated for each wave length and ship speed. If these values are treated statistically by the method outlined above for amplitudes of motion, the frequency of deck submergence or forefoot emergence in an irregular sea can be estimated. The frequencies thus obtained in one case were found to agree well with observations in a towing tank irregular wave test. Since the ship motions derived analytically here for hulls of normal form are close to the model test data, the calculated motions can be used instead of model tests to predict bow submergence, hence shipping of water, and frequency of forefoot emergence, which would qualitatively indicate likelihood of slamming.

It is well known, however, that out of a number of forefoot emergences only a few result in slamming. In order to predict quantitatively the frequency of slamming a more elaborate statistical treatment of either experimental or calculated responses to regular waves is needed. The probability must be found of the joint occurrence of forefoot emergence,

a high descending velocity of the bow and approximate tangency of the keel to the water surface at the instant of impact. A beginning has been made in the work of L. J. Tick (1954) where the probability of the joint occurrence of two of the conditions, forefoot emergence and high descending velocity, was investigated.

It is believed that the phase relationships are of greatest importance in this connection. Sea captains and other observers are aware of the difference small changes of speed make in the shipping of water and slamming. With propeller R.P.M. on a typical cargo ship reduced to say 75 in mildly adverse conditions, the change of 5 R.P.M. often makes the difference between acceptable and unsatisfactory ship behavior. There is little change in amplitude of motion in this case and the change in ship behavior appears to be primarily due to change in phase relationship. At the lower speed the descending ship bow may settle quietly into the hollow of a wave and at 5 R.P.M. more it may strike with force the flank of an oncoming wave, causing either slamming or shipping of water or both in succession.

Since the phase relationships in regular waves are given satisfactorily by the method of calculation presented here, it can be expected that this behavior will be predicted correctly. Thus the stage has been reached at which the significant aspects of ship performance at sea can be predicted by analytical methods.

CONCLUDING REMARKS

At the conclusion of Korvin-Kroukovsky's 1955 paper, two lines of research were suggested to improve the theory of ship motions in regular waves. These were 1) the solution of non-linear equations of motion (using numerical methods of computation if necessary) and 2) the experimental evaluation of the various coefficients of the equations of motion. The need for such research appeared to be acute since there was a large systematic error in the analytically computed phase lags. However, it has been shown, most of this error was due to omission of certain dynamic coupling terms. Application of the corrected linear theory described in the present paper has yielded generally satisfactory estimates of both amplitudes and phase relationships for ships of normal commercial form. Apparently the deviations of most of these ship forms from the simplifying assumptions of the theory are not large enough to affect the results seriously.

While the suggested research is no longer critically necessary it is still desirable. The present theory has failed in application to a rather unusual ship form (the yacht and its lengthened counterpart) characterized by sloping sides at the LWL throughout its length and by a pronounced bow overhang and a cutaway forefoot. Development of the theory along the lines proposed is absolutely necessary for success in

estimating the behavior of such hulls and would make the theory more reliable for ship types like the Maier form and the fishing trawlers.

Since publication of the theory of ship motions in irregular seas by St. Denis and Pierson (1953), a large amount of practical experience in its application has been accumulated by E. V. Lewis (1955, 1957), Lewis and Numata (1957), and Lewis and Dalzell (1957) in connection with towing tank experiments, and by Cartwright and Rydill (1957) and Cartwright (1957) in connection with an actual ship at sea. The validity and usefulness of this statistical theory have now been fully established. The theory of motions in regular waves can therefore no longer be considered by itself alone. It is only a part of the picture, the hydro-mechanical phase which establishes the dependence of a ship's motions on its form and mass distribution. The results obtained in this part, the ship responses to regular waves or "response factors", are then treated by methods of mathematical statistics in conjunction with a measured or assumed spectrum of a realistic irregular sea to give the realistic ship motions.

These statistical methods assume that the motions vary linearly with wave height and require that the results of the hydro-mechanical phase be supplied in linear form as ratios of response to wave height. The recommendation that solutions of non-linear equations of motion be

obtained should thus be qualified. What is desired is not a rigorous solution of a non-linear problem but rather a solution of a substitute linear problem which would approximate the true solution. Such a process of "equivalent linearization" has been used by Golovato (1956) to take into account the effects of non-linear damping in heave. However, non-linearities are usually pronounced in both the damping force coefficients and the restoring force coefficients, and the coupling of heaving and pitching motions further aggravates the difficulties of this problem.

It is to be noted, also, that in the statistical method the ship responses to a number of wave lengths are averaged, so that non-systematic errors tend to compensate each other. Because of this, and the statistical uncertainties in the evaluation of a sea spectrum as well, there is less need for a high degree of accuracy in estimating the ship response. The theory of ship motions can thus in its present state of development be practically useful. The application of the theory is not limited to amplitudes of motions and accelerations; the frequency of shipping water can be readily predicted and prediction of the frequency of slamming is not too far in the future.

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NOMENCLATURE

The following nomenclature is used throughout the paper:

- A, B, C,) = Coefficients of miscellaneous terms of the differential
D, E, G) equations of motions, Equations (2).
- a, b, c,) = Coefficients of miscellaneous terms of the differential
d, e, f) equations of motions, Equations (2).
- \bar{A} = Ratio of amplitude of waves made by ship to amplitude
 of heaving motion.
- a_0 = Amplitude of vertical acceleration at the bow.
- B = Beam (local).
- b, s = Instantaneous distances of ship bow and stern from the
 nodal point of wave as defined in Fig. 14.
- c = Wave celerity.
- \bar{E} = Area under spectrum.
- F = Hydrodynamic heaving force.

- \bar{F} = Heaving force imposed on a ship by waves ($= F_0 e^{i\sigma}$).
- g = Acceleration of gravity.
- H_f = Force due to water pressures generated by waves and the ship's oscillations.
- H_m = Moment about c.g. due to water pressures generated by waves and ship's oscillations.
- h = Wave amplitude.
- J = Longitudinal moment of inertia of a ship in mass units.
- K_1, \bar{A}_2 = Coefficients of Equation (25).
- k_2 = Added mass coefficient in two-dimensional vertical flow about a ship section.
- k_4 = Correction coefficient for effect of free water surface.
- L = Ship length.
- M = Hydrodynamic moment.
- \bar{M} = Pitching moment imposed on a ship by waves ($= M_0 e^{i\tau}$).
- m = Mass of a ship or of a ship section.
- $N(\xi)$ = Vertical damping force per unit of body length per foot per second.
- P, Q, R, S = Groupings of coefficients of the differential equations of motions defined by Equations (7).

p	= Pressure.
R	= Radial distance to a point q in fluid.
r	= Local radius of semi-cylindrical body.
S	= Sectional area.
s_o	= Amplitude of vertical displacement at the bow.
t	= Time.
u	= Horizontal component of orbital velocity of water in waves.
V	= Ship speed.
v	= Vertical velocity.
v_w	= Vertical component of wave orbital velocity.
x	= Longitudinal co-ordinate with respect to wave nodal point.
y	= Vertical co-ordinate or local half-breadth of LWL plane.
\bar{z}	= Complex amplitude of heaving motion ($= Z_o e^{i\delta}$).
z	= Vertical co-ordinate or heaving displacement.
α	= Polar co-ordinate.
β	= Angle between longitudinal tangent to body surface and x -axis.
δ, ϵ	= Phase angles of ship motions.
η	= Local wave-height co-ordinate.

- θ = Angle of pitch.
- $\bar{\theta}$ = Complex amplitude of pitching motion ($= \theta_0 e^{i\epsilon}$).
- λ = Wave length.
- ξ = Longitudinal co-ordinate with respect to CC.
- ρ = Water density.
- σ, τ = Phase angles of exciting forces.
- ϕ = Velocity potential.
- ω or ω_e = Frequency of wave encounter.

A P P E N D I X

EVALUATION OF HYDRODYNAMIC FORCES

This Appendix is arranged to follow as closely as possible Appendix I of the earlier paper (Korvin-Kroukovsky, 1955b) so that the changes made can be easily seen. Where sufficient discussion was given in that reference, the details will be omitted here.

Formulation of the Problem

Consider a ship moving with a constant forward velocity V (i.e. neglecting surging motion) with a train of regular waves of celerity c . Assume the set of coordinate axes fixed in the undisturbed water surface,

with the origin instantaneously located at the wave nodal point preceding the wave rise, as shown in Fig. 14. With increase in time t the axes remain fixed in space, so that the water surface rises and falls in relation to them. This vertical displacement at any instant and at any distance x is designated η . Imagine two control planes spaced dx apart at a distance x from the origin, and assume that the ship and water with orbital velocities of wave motion penetrate these control surfaces. Assume that the perturbation velocities due to the presence of the body are confined to the two-dimensional flow between control planes, i.e. neglect the fore-and-aft components of the perturbation velocities due to the body, as in the "slender body theory" of aerodynamics. This form of analysis, also known as the "strip method" or "cross flow hypothesis", is thus an approximate one in the sense that a certain degree of interaction between adjacent sections is neglected.

The cross section of the ship at x will now be taken as semi-circular; the correction necessary to represent other ship sections will be introduced later. Following F.M. Lewis (1929) and Weinblum and St. Denis (1950), the flow about the semi-submerged body used in the basic derivation will be assumed to be identical with that about the lower half of a fully submerged body. Corrections to account for the presence of the free water surface will be brought in later.

In considering the pitching and heaving motions of the body it is necessary to introduce a second coordinate system moving with the ship with its origin at the center of gravity of the ship. The longitudinal distance of any section of the ship from the origin is designated ξ (positive forward). Vertical displacement of the C.G. (i.e. the heave) is designated by z (positive upwards) and angular displacement or pitching motion is designated by θ (positive for bow displaced upwards). The vertical displacement of the section at x due to pitching is then $\xi \theta$, with θ in radians, for the relatively small angles encountered. (It is also assumed that $\cos \theta \approx 1$.)

The two-dimensional flow pattern between the control planes at x results from three imposed motions:

1. Vertical velocity of the center of the circle

$$v = \dot{z} + \xi \dot{\theta} - v \theta. \quad (8)$$

2. Vertical component of wave orbital velocity

$$v_w = \dot{\eta} e^{2\pi y/\lambda} = \frac{-2\pi h c}{\lambda} e^{2\pi y/\lambda} \cos \frac{2\pi}{\lambda} (x - ct) \quad (9)$$

where h is wave amplitude, λ is wave length, $y = -R \cos \alpha$, the depth below the still-water level to any point in the fluid, and $\eta = h \sin 2\pi (x - ct)/\lambda$.

3. Apparent variation of the radius r of the ship section at the control planes with time; $r = r(t)$.

All motions are assumed to be sufficiently small so that the derivatives of the potential can be taken on the surface of the circle as if its center were at its initial position $y = 0$, and the known expression for the potential in a uniform fluid stream can be applied, despite the slight non-uniformity induced by the waves which are assumed to be small.

The vertical hydrodynamic force acting on the length dx of the submerged semi-cylinder is given by

$$\frac{dF}{dx} = 2r \int_0^{\pi/2} p \cos \alpha \, d\alpha \quad (10)$$

where F denotes the vertical force, α is the polar angle as defined in Fig. 1b, and the time-dependent part of the pressure, p , is obtained from Bernoulli's equation. Neglecting the squares of small perturbation velocities

$$p = \rho \frac{\partial \phi}{\partial t} \quad (11)$$

where ϕ is the velocity potential and ρ the mass density.

The velocity potential of the flow about a cylinder due to the relative vertical velocity $(v - v_w)$ is given by

$$\phi_b = -(v - v_w) \frac{r^2}{R} \cos \alpha. \quad (12)$$

The first term of (12) may be considered as the potential due to the body motion in smooth water, designated ϕ_{bm} ,

$$\phi_{bm} = -v \frac{r^2}{R} \cos \alpha. \quad (13)$$

The second term of (12) is the potential due to body-wave interaction, ϕ_{bw} :

$$\begin{aligned} \phi_{bw} &= v_w \frac{r^2}{R} \cos \alpha. \\ &= -\frac{2\pi h c}{\lambda} \frac{r^2}{R} e^{2\pi y/\lambda} \cos \alpha \cos \frac{2\pi}{\lambda} (x - ct). \end{aligned} \quad (14)$$

The potential due to wave motion alone is ϕ_w ,

$$\phi_w = h c e^{2\pi y/\lambda} \cos \frac{2\pi}{\lambda} (x - ct). \quad (15)$$

The total velocity potential is the sum of Equations (13), (14) and (15),

$$\phi = \phi_{bm} + \phi_{bw} + \phi_w. \quad (16)$$

Exciting Forces

Attention will now be concentrated on the second and third terms of Equation (16), the two component parts of the velocity potential which give the exciting forces due to waves.

The pressure due to ϕ_{bw} is, from (11) and (14),

$$\begin{aligned} P_{bw} = \rho \frac{\partial \phi_{bw}}{\partial t} &= -\frac{4\pi^2 c^2 h}{\lambda^2} \rho \frac{r^2}{R} e^{(-2\pi R \cos \alpha)/\lambda} \cos \alpha \sin \frac{2\pi}{\lambda} (x - ct) \\ &\quad - \frac{2r}{R} \rho \frac{2\pi h c}{\lambda} e^{(-2\pi R \cos \alpha)/\lambda} \cos \alpha \cos \frac{2\pi}{\lambda} (x - ct). \end{aligned}$$

On the surface of the body where $R = r$ (and since $c^2 = g \lambda / 2\pi$),

$$p_{bw} = - \left[\frac{2\pi}{\lambda} \rho g h \sin \frac{2\pi}{\lambda} (x - ct) \right] r \cos \alpha e^{(-2\pi r \cos \alpha)/\lambda} \\ - \left[\frac{2\rho g h}{c} \cos \frac{2\pi}{\lambda} (x - ct) \right] \dot{r} \cos \alpha e^{(-2\pi r \cos \alpha)/\lambda} \quad (17)$$

(In the 1955 paper an error was made in substituting $R = r$ before differentiating with respect to time, since r is a function of time.)

The corresponding component of the force due to body-wave interference is obtained by substitution of (17) in Equation (10):

$$\left(\frac{dF}{dx} \right)_{bw} = \left\{ -4 \frac{\pi}{\lambda} \rho g h r^2 \sin \frac{2\pi}{\lambda} (x - ct) - 4 \rho \frac{g h r \dot{r}}{c} \cos \frac{2\pi}{\lambda} (x - ct) \right\} \\ \times \int_0^{\frac{\pi}{2}} \cos^2 \alpha e^{(-2\pi r \cos \alpha)/\lambda} d\alpha$$

The series expansion of the exponential is

$$e^{(-2\pi r \cos \alpha)/\lambda} = 1 - \frac{2\pi r}{\lambda} \cos \alpha + \frac{2\pi^2 r^2 \cos^2 \alpha}{\lambda^2} \dots$$

and the integral is evaluated as

$$\int_0^{\frac{\pi}{2}} \cos^2 \alpha e^{(-2\pi r \cos \alpha)/\lambda} d\alpha = \frac{\pi}{4} - \frac{4\pi r}{3\lambda} + \frac{3\pi^3 r^2}{8\lambda^2} \dots$$

Neglecting cubes and higher powers of the small quantity r/λ in the first term and also r^2/λ^2 in the coefficient of the \dot{r} term,

$$\left(\frac{dF}{dx} \right)_{bw} = -2 \rho g h r \left\{ \left(\frac{\pi^2 r}{2\lambda} - \frac{8\pi^2 r^2}{3\lambda^2} \right) \sin \frac{2\pi}{\lambda} (x - ct) \right. \\ \left. + \frac{\dot{r}}{c} \left(\frac{\pi}{2} - \frac{8\pi r}{3\lambda} \right) \cos \frac{2\pi}{\lambda} (x - ct) \right\} \quad (18)$$

The pressure due to the potential of wave motion ϕ_w is,

from (11) and (15),

$$p_w = \rho \frac{\partial \phi_w}{\partial t} = \frac{2 \pi \rho h c^2}{\lambda} e^{2\pi y/\lambda} \sin \frac{2\pi}{\lambda} (x - ct).$$

At the surface of the body,

$$p_w = \rho g h e^{(-2\pi r \cos \alpha)/\lambda} \sin \frac{2\pi}{\lambda} (x - ct). \quad (19)$$

The corresponding component of the force is obtained by substituting

(19) in (10):

$$\left(\frac{dF}{dx}\right)_w = 2 \rho g h r \sin \frac{2\pi}{\lambda} (x - ct) \int_0^{\pi/2} \cos \alpha e^{(-2\pi r \cos \alpha)/\lambda} d\alpha$$

where the integral is equal to

$$1 - \frac{\pi^2 r^2}{2 \lambda} + \frac{4 \pi^2 r^2}{3 \lambda^2} \dots$$

Again neglecting cubes and higher powers of r/λ :

$$\left(\frac{dF}{dx}\right)_w = 2 \rho g h r \left(1 - \frac{\pi^2 r^2}{2 \lambda} + \frac{4 \pi^2 r^2}{3 \lambda^2}\right) \sin \frac{2\pi}{\lambda} (x - ct). \quad (20)$$

The first term of Equation (20) is seen to be the displacement force resulting from the wave rise or fall and the accompanying increase or decrease of volume. The second and third terms represent a modification of this force, due to the (approximate) exponential variation of pressure with depth, and this modification is known as the "Smith Effect". The entire equation represents the force acting under what is usually referred to as the "Froude-Kriloff Hypothesis". This force which is

exerted by the waves on the body is reduced by the body-wave interference effect indicated by Equation (18); for a semi-cylindrical section at zero speed this amounts to approximately doubling the "Smith Effect".

Designating by β the angle between the longitudinal tangent to the body surface and the positive x-axis, the derivative \dot{r} is evaluated as

$$\dot{r} = \frac{dr}{d\xi} \frac{d\xi}{dt} = -V \tan \beta. \quad (21)$$

(In the 1955 paper the sign was erroneously taken as positive.) With the substitution of (21), the sum of Equations (18) and (20), which is the total exciting force, becomes

$$\begin{aligned} \frac{dF}{dx} &= \left(\frac{dF}{dx}\right)_w + \left(\frac{dF}{dx}\right)_{bw} \\ &= 2 \rho g h r \left\{ \left(1 - \frac{\pi^2 r}{\lambda} + \frac{4 \pi^2 r^2}{\lambda^2}\right) \sin \frac{2\pi}{\lambda} (x - ct) \right. \\ &\quad \left. + \frac{V}{c} \tan \beta \left(\frac{\pi}{2} - \frac{8 \pi r}{3 \lambda}\right) \cos \frac{2\pi}{\lambda} (x - ct) \right\} \quad (22) \end{aligned}$$

This expression replaces Equation 26 of the 1955 paper. It differs from it in the sign of the velocity-dependent terms, which are small, and in the value of the coefficient of the $(r \tan \beta)/\lambda$ term, which is also small.

Two steps remain to be taken. First, Equation (22) must be generalized for ship sections other than semi-circular and second, corrections must be introduced for free surface effects.

It is clear that the factor $2 \rho g r h \sin 2\pi (x - ct)/\lambda = 2 \rho g r \eta$ represents the change in displacement force with wave rise and fall; r in this case then is to be taken as the half-beam, $B/2$, at the load water-line. Next it is noted that when the body-wave interaction is taken into account the "Smith Effect" on a circular body is doubled. This factor of 2 may be interpreted as $(1 + k_2)$ by analogy with G.I. Taylor's expression (1928) for the force acting on a body placed in a fluid flow with a velocity gradient. Here k_2 is the coefficient of accession to inertia in vertical flow and is equal to 1 for a circular section.

With a free water surface and the formation of a standing wave system, the value of $k_2 = 1$ for the circular cylinder is modified by a factor which is designated as k_4 . Ursell (1954, and answer to Discussion, Korvin-Kroukovsky, 1955b) has computed the following values of k_4 versus $\omega^2 r/g$ (or $\omega^2 B/2g$) for the circular cylinder:

$\frac{\omega^2 B/2g}{0}$	$\frac{k_4}{\infty}$
.262	.818
.524	.632
.785	.592
1.571	.673
2.094	.738
2.356	.762
3.142	.818
3.927	.859
4.712	.883

In the absence of more complete information it will be assumed that this table of corrections applies to non-circular sections as well.

From experiments with an oscillator, Golovato (1956) derived the coefficients of added (virtual) mass in heaving oscillation for a ship form symmetrical fore-and-aft with U-sections almost wall-sided at the load waterline. Fig. 8 of that reference shows a curve very similar in trend to the coefficients of the above table but with values about 20% higher and with the minimum shifted to a somewhat higher frequency. The effect of these differences on the ship response are expected to be small.

The factor $(1 + k_2 k_h)/2$ will then be applied to all terms of Equation (22) after the first displacement force term. In the earlier paper, k_h was estimated for the ship as a whole and it was applied only to the integrated virtual mass and inertia effects due to body motion in smooth water; it was omitted in the expression for the exciting forces. Subsequently, it was found necessary to apply the k_h correction to each section for the calculation of bending moments and highly advisable to adopt this more accurate procedure for the motion calculations. This omission has therefore been corrected in the present paper.

Since the modified "Smith Effect" terms are connected with virtual masses and since the effect of section shape is defined by the coefficient k_2 , the factor r in this case is interpreted as a measure of sectional area: i. e.

$$r = \sqrt{2 S/\pi} \quad (23)$$

where S is sectional area below the load waterline, and therefore

$$\tan \beta = \frac{d r}{d \xi} = \frac{1}{\sqrt{2 \pi S}} \frac{d S}{d \xi} \quad (24)$$

With the substitutions indicated above, the distribution of vertical heaving forces due to the action of waves on a ship at a particular instantaneous position of the ship on the wave ($t = 0$) is expressed as

$$\frac{d F}{d x} = \rho g h B (K_1 \sin \frac{2 \pi x}{\lambda} + K_2 \cos \frac{2 \pi x}{\lambda}) \quad (25)$$

where K_1 and K_2 are non-dimensional coefficients:

$$K_1 = 1 - \frac{1 + k_2 k_h}{2} \frac{\pi^2}{\lambda} \sqrt{\frac{2 S}{\pi}} + h(1 + k_2 k_h) \frac{\pi}{\lambda^2} S,$$

$$K_2 = \frac{\pi(1 + k_2 k_h)}{4} (1 - \frac{16}{3\lambda} \sqrt{\frac{2 S}{\pi}}) \frac{V}{c} \frac{1}{\sqrt{2 \pi S}} \frac{d S}{d \xi}.$$

These coefficients depend on the sectional shape and area, on the wave length and also, because of the presence of the coefficient k_h , on the frequency of wave encounter. The distribution of forces along the length of the ship given by Equation (25) can be used directly in the computation of the bending moments exerted on a ship by waves.

For an analysis of ship motions the force distribution must be integrated to provide the total heaving force F and the total pitching moment M :

$$F = \int_a^b \frac{d F}{d x} d x \quad (26)$$

and

$$M = \int_s^b \frac{dF}{dx} \left(\xi + \frac{1}{n} \frac{dS}{d\xi} \right) dx \quad (27)$$

where the limits of integration s and b are the values of x at the stern and the bow, respectively. The second term of Equation (27) results from the consideration that the water pressure acts normal to the body surface; in the case of a body of varying circular section the moment arm is $(\xi + r \tan \beta)$ and, by the use of the relationships (23) and (24), for a normal ship form the moment arm may be assumed to be $(\xi + dS/n d\xi)$. Equations (25), (26) and (27) replace Equations (33) and (34) of the earlier paper.

The integrals of (26) and (27) can be evaluated readily by Simpson's rule. By changing the ship's position relative to the wave the maximum values or amplitudes of the exciting force and moment can be found as well as the phase lags σ and τ . Calculations of these amplitudes, F_0 and M_0 , were made for a 5-ft-long model of the Series 60, 0.60 block coefficient hull (ETT Model No. 1445) in waves of ship length $(\lambda/L = 1)$ and wave height of 1.5 in., for comparison with the experimental data described in Appendix 2 of the 1955 paper. A very good agreement between calculated and experimental values was obtained except at zero and very low forward speeds (see Fig. 15). It has been observed in recent years that at such speeds there is often interference

from the waves reflected by the walls of the towing tank, while at greater speeds this interference is no longer encountered and the experimental data become more reliable.

Forces Due to Body Motions

The pressure in the fluid due to the body's own motion is, from Equations (11) and (13),

$$\begin{aligned} p_{bm} &= \rho \frac{\partial \phi_{bm}}{\partial t} = -\rho \dot{r} \frac{r^2}{R} \cos \alpha - \frac{2\rho v r \dot{r}}{R} \cos \alpha \\ &= -\rho \dot{r} r \cos \alpha - 2\rho v \dot{r} \cos \alpha \end{aligned} \quad (28)$$

at the body surface where $R = r$. The distribution of vertical forces is obtained by substituting (28) in (10):

$$\left(\frac{dF}{dx} \right)_{bm} = -\rho \frac{\pi}{2} r^2 \dot{r} - \rho \pi r \dot{r} v \quad (29)$$

The apparent vertical velocity v of the center of the circular section, given by Equation (8), consists of three parts

$$v = \dot{z} + \xi \dot{\theta} - V \theta,$$

where the first is the heaving velocity, the second is the vertical velocity due to the angular velocity of pitching and the third is the vertical velocity due to the instantaneous angle of trim θ at the control planes. Since ξ is a function of time and $\dot{\xi} = -V$,

$$\dot{v} = \ddot{z} + \xi \ddot{\theta} - 2V \dot{\theta}. \quad (30)$$

After substituting (8), (21), and (30) in Equation (29), the following

set of six terms is obtained:

$$\begin{aligned}
 \left(\frac{dR}{dx}\right)_{bm} &= - \left[\rho \frac{\pi}{2} r^2 \right] \ddot{z} \dots\dots\dots(1) \\
 &- \left[\rho \frac{\pi}{2} r^2 \xi \right] \ddot{\theta} \dots\dots\dots(2) \\
 &+ 2 \left[\rho \frac{\pi}{2} r^2 v \right] \dot{\theta} \dots\dots\dots(3) \\
 &+ \left[\rho \pi r \tan \beta v \right] \ddot{z} \dots\dots\dots(4) \\
 &+ \left[\rho \pi r \tan \beta v \xi \right] \dot{\theta} \dots\dots\dots(5) \\
 &- \left[\rho \pi r v^2 \tan \beta \right] \theta \dots\dots\dots(6)
 \end{aligned}
 \tag{31}$$

Equation (31) replaces Equation (39) of the 1955 paper. Terms (1) and (2) are identical with (3) and (6) of Equations (39) and term (3) is the sum of the earlier (1) and (5) with the sign of the latter corrected. Terms (4), (5) and (6) are twice terms (4), (7) and (2) respectively of the earlier Equation (39), which were incorrect because substitution of $R = r$ had been made before differentiation with respect to time.

The factor $(\rho \pi r^2/2)$ in the first three terms of (31) is evidently the virtual mass of an element of body length, and by introducing k_2 and k_4 on the basis of the reasoning outlined in connection with exciting forces, it can be expressed as $(\rho S k_2 k_4)$. The factor $(\rho \pi r \tan \beta)$ in the last three terms is the derivative with respect to ξ of $(\rho \pi r^2/2)$ and so it can be expressed as $d(\rho S k_2 k_4)/d\xi$. The total

force due to the body's own motion is then

$$F_{bm} = -\rho \int S k_2 k_u (\ddot{z} + \xi \ddot{\theta} - 2 \nabla \dot{\theta}) d\xi$$

$$+ \nabla \rho \int \frac{d S k_2 k_u}{d \xi} (\dot{z} + \xi \dot{\theta} - \nabla \theta) d\xi$$
(32)

and the moment is

$$M_{bm} = -\rho \int S k_2 k_u (\ddot{z} + \xi \ddot{\theta} - 2 \nabla \dot{\theta}) \xi d\xi$$

$$+ \nabla \rho \int \frac{d S k_2 k_u}{d \xi} (\dot{z} + \xi \dot{\theta} - \nabla \theta) \xi d\xi$$
(33)

where the integration is carried over the length of the hull.

Since F_{bm} and M_{bm} are functions of z and θ and their derivatives, the terms of Equations (32) and (33) may be transposed to the left-hand side of the coupled equations of motion. Their contributions to the coefficients of these equations are designated by the subscript 1. Thus

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$$a_1 = \rho \int (S k_2 k_u) d\xi$$

$$b_1 = -v_0 \int \frac{d(S k_2 k_u)}{d\xi} d\xi = 0$$

$$d_1 = \rho \int (S k_2 k_u) \xi d\xi$$

$$e_1 = -2 v_0 \int (S k_2 k_u) d\xi - v_0 \int \frac{d(S k_2 k_u)}{d\xi} \xi d\xi$$

$$f_1 = +v_0^2 \int \frac{d(S k_2 k_u)}{d\xi} d\xi = 0$$

$$A_1 = \rho \int (S k_2 k_u) \xi^2 d\xi$$

$$B_1 = -2 v_0 \int (S k_2 k_u) \xi d\xi - v_0 \int \frac{d(S k_2 k_u)}{d\xi} \xi^2 d\xi$$

$$c_1 = v_0^2 \int \frac{d(S k_2 k_u)}{d\xi} \xi d\xi$$

$$n_1 = \rho \int (S k_2 k_u) \xi d\xi$$

$$R_1 = -v_0 \int \frac{d(S k_2 k_u)}{d\xi} \xi d\xi$$

Dynamic Terms in $\dot{\mathbf{z}}$ and $\dot{\mathbf{e}}$

Attention should be called to the fact that the velocity-dependent terms in $\dot{\mathbf{z}}$ and $\dot{\mathbf{e}}$ of Equations (31), (32) and (33) do not involve dissipa-

portion of energy, but only the transfer of energy from one mode to another. This was demonstrated by Bicknell (1946) and by Havelock (1955) who refers to these terms as "dynamic coupling".

In previous studies if oscillations velocity-dependent terms appear only in the role of energy dissipation either by viscosity or by wave-making in the case of ships. Following these earlier studies it was assumed in the 1955 paper that the velocity-dependent terms in the development of the potential theory merely implied damping and could be replaced by damping terms determined on the basis of energy dissipation by waves in a *quid pro quo*, since the initial statement of the problem did not provide for inclusion of energy dissipation terms. Later, examination of the work of Fey (1957) suggested that this was an error of judgment which could be responsible for the poor correlation between calculated and experimental phase relationships reported in the earlier paper and also for the poor results obtained when applying the method of that reference to the calculation of bending moments.

A study of Bicknell (1946) and Havelock (1955) confirmed this and therefore the terms in $\dot{\eta}$ and $\dot{\theta}$, (3), (4) and (5) of Equation (2) have been reinstated. They yield a heaving force due to pitching velocity $\dot{\theta}$ and a pitching moment due to heave velocity $\dot{\eta}$ equal to

$$\left[-2\rho \int (S k_2 k_4) d\xi - \rho \int \xi d(S k_2 k_4) \right] V \dot{\theta}$$

and

$$\left[-\rho \int \xi d(S k_2 k_4) \right] V \dot{z} .$$

(35)

For the case of a half-immersed spheroid under the condition of a free water surface but neglecting wave-making, Havelock gives the dynamic coupling terms as

$$- p M V \dot{\theta} \text{ for the heaving force}$$

$$\text{and} \quad + q M V \dot{z} \text{ for the pitching moment,}$$

where M is the mass of displaced water. In general $p \neq q$, and each is given by a fairly complicated expression in terms of ellipsoidal coordinates and associated Legendre functions of the second kind. For a long spheroid Havelock finds that $p = q = (1 + k_1)/2$, or .515 for a length-diameter ratio of 8 which is the fineness ratio of the usual ship form. (Haskind (1946) had found that $p = q$ for a thin or "Michell" ship symmetrical fore-and-aft.)

For a prolate spheroid S is a function of ξ alone and

$$\int_L S d\xi = - \int_L \xi dS = M/\rho .$$

Therefore expression (35) of the present development becomes

$$- \overline{k_2 k_4} M V \dot{\theta}$$

and

$$+ \overline{k_2 k_4} M V \dot{z}, \text{ where the bar indicates the value for the entire body.}$$

Thus $p = q = \overline{k_2 k_4}$. For a circular section $k_2 = 1$ and, at the oscillating frequency in the vicinity of synchronism for most ship forms, $\overline{k_4}$ is of the order of 0.75. It appears from the application to the models in this paper that the damping in heave is reduced and in pitch is increased by the addition of the dynamic coupling terms.

Dissipative Damping

It was mentioned previously that the free water surface was not taken into account in the basic derivation of the present paper and that a correction for it must be introduced independently. The effect of the free surface on the virtual mass has been allowed for approximately by the use of the coefficient k_4 derived by Ursell for a semi-cylinder (1954, and Discussion, Korvin-Kroukovsky, 1955b). (Grim, 1953, has also calculated this effect for some ship-like forms but his material is not extensive enough for general application.) The other well known effect of the free surface is the dissipation of energy in the formation of waves which propagate away from the ship in all directions. In the "strip" method of analysis it is assumed that waves from each length segment $d\xi$ propagate laterally. If the ratio of the amplitude of these waves to the amplitude of the heaving motion of a ship section is designated by \bar{A} , the damping force per unit vertical velocity v of the ship segment is expressed as (Holstein 1936, 1937a, 1937b, and

Havelock (1942)

$$\bar{N}(\xi) = \rho g^2 \bar{A}^2 / \omega_e^3, \quad (36)$$

where ω_e is the frequency of the waves radiated by the ship and is equal to the frequency of wave encounter.

Holstein (1936, 1937a and 1937b) and Havelock (1942) represented the heaving body by a distribution of harmonically pulsating sources along the bottom. In that case the amplitude ratio \bar{A} is given by

$$\bar{A} = 2 e^{-k_0 f} \sin(k_0 y) \quad (37)$$

where $k_0 = \omega_e^2 / g$, y is the half-beam $B/2$, and $f = S/B$, the mean draft of a ship section. This theoretical result was confirmed by Holstein's own experiments; the results of the many experiments appear consistent, yet some doubt may be felt because of the smallness of the test tank. The theory is approximate and acceptance of it necessarily hinges on agreement with experimental results.

A more nearly exact theory was developed by Ursell (1949, 1953, 1954) for a heaving semicylinder, and by Grim (1953) for a number of analytically defined sections closely approaching practical ship sections. In the case of a semicircular section Grim's results agree with Ursell's. In their theory the damping force is calculated as a boundary value problem. Unfortunately no experimental verification has been

provided.

Both theories, that of Holstein and Havelock which does not satisfy the boundary condition on the surface of the body and that of Grim which fulfills all the boundary conditions with a good degree of accuracy, give approximately the same results at the frequency for synchronization for a normal ship, but very different results at higher frequencies. The Holstein-Havelock values of \bar{A} , Equation (37), were used in the 1955 paper where the results of the calculations generally indicated over-damping. In recent work Grim's \bar{A} values, which are presented in the form of charts, have been substituted. This has given a better correlation of the calculated and experimental amplitudes of the motions of the ship models.

The experiments of Colovato (1956) have already been mentioned in connection with coefficients of added mass. In those experiments the coefficient b of damping force was also measured. The experimentally measured b was found to be lower than the coefficients computed by the different methods of Havelock and Grim, that computed by Grim's method being closer to the measured value. Another comparison by Colovato of computed and measured coefficients using the experimental data of Haskind and Raman (1946) gave similar results.

The computed coefficients are based on the "strip" method of analysis which assumes two-dimensional fluid flow, while Golovato's and Haskind's and Riman's tests were made with ship models, i.e. in three-dimensional flow. The relation of damping in three-dimensional flow to damping in two-dimensional flow can be estimated on the basis of the work of Havelock (1956) and of Vossers (1956). Havelock calculated the damping coefficient by two methods, three-dimensional and "strip", for a submerged spheroid with length-beam ratio of 8, a fair value for comparison with ship models. Vossers made similar computations from Haskind's theoretical results (1946) for a "thin ship" in the sense of the approximations introduced by Michell (1896) in his theory of wave resistance of ships. The results of both are reproduced here in Figs. 16, 17 as ratios of the coefficients of damping in heave or pitch by the two methods.¹⁰ It should be remembered that accurate evaluation of damping is most important in the vicinity of synchronism. The values of the frequency parameter $\omega^2 L/g$ ($\sigma^2 L/g$ in the figures) at synchronism are given in the last two columns of Table 5 for the eight models to which the computational method outlined here has been applied. (Also included in Table 5 are the parameters L/B , k_2 and k' for comparison with Havelock's submerged spheroid.)

¹⁰ The subscript S in the figures denotes "strip" method.

Omitting from consideration the original trawler and yacht, the lowest values of the parameter $\omega^2 L/g$ are 10.4 in heave for the T-2 tanker and 11.7 in pitch for the Series 60 hull, while for the models with displacement-length ratio of 60 $\omega^2 L/g$ is above 17.6. At these values Figs. 16 and 17 indicate negligible corrections for damping in heave and small corrections for damping in pitch (0 to 15% based on Havelock's computations and 10 to 20% from Vossers'). Since the corrections for three-dimensional effect are negligible or small and since they have not yet been developed for ships of normal form, they will be ignored in the present work; the damping will be taken as computed for two-dimensional flow by Grim. It is gratifying nevertheless to have measures of possible error such as Figs. 16 and 17 to replace arbitrary assumptions made in the earlier paper as to the applicability of the "strip" method of analysis.

The damping force for each section is

$$v N(\xi) = \left[\dot{z} + \xi \dot{\theta} - v \theta \right] N(\xi), \quad (38)$$

where $N(\xi)$, the damping force per unit vertical velocity of the ship section, is given by (36). The total damping force and moment of the hull are then obtained by integration over the length, thus the damping force is

$$\int N(\xi) \left[\dot{z} + \xi \dot{\theta} - v \theta \right] d\xi \quad (39)$$

and the damping moment is

$$\int N(\xi) \left[\dot{z} + \xi \dot{\theta} - \gamma \theta \right] \xi d\xi. \quad (4C)$$

The contributions of these expressions to the coefficients of the coupled equations of motion are designated by the subscript 2:

$$b_2 = \int N(\xi) d\xi$$

$$e_2 = E_2 = \int N(\xi) \xi d\xi$$

$$g_2 = -\gamma \int N(\xi) d\xi = \gamma b_2 \quad (4D)$$

$$B_2 = \int N(\xi) \xi^2 d\xi$$

$$C_2 = -\gamma \int N(\xi) \xi d\xi$$

The integrations are carried out numerically as in all cases in this paper, and over the length of the hull.

When all the forces and moments proportional to z and θ and their derivatives with respect to time, which are presented in the above development of the potential theory with corrections for free surface effects, are combined with the inertial and restoring forces and moments the coefficients of the equations of motion become

$$a = \pi + \rho \int (S \kappa_2 \kappa_3) d\xi$$

$$A = \rho \int (S \kappa_2 \kappa_3) \xi^2 d\xi$$

$$d = D = \rho \int (S \kappa_2 \kappa_3) \xi d\xi$$

$$b = \int Y(\xi) d\xi$$

$$B = \int Y(\xi) \xi^2 d\xi - 2\pi D - \pi \rho \int \xi^2 d(S \kappa_2 \kappa_3)$$

$$e = \int Y(\xi) \xi d\xi - 2\pi C \int (S \kappa_2 \kappa_3) d\xi - \pi \rho \int \xi d(S \kappa_2 \kappa_3)$$

$$E = \int Y(\xi) \xi d\xi - \pi \rho \int \xi d(S \kappa_2 \kappa_3) \quad (12)$$

$$c = \rho g \int B d\xi$$

$$C = \rho g \int B \xi^2 d\xi - \pi E$$

$$g = \rho g \int B \xi d\xi - \pi C$$

$$G = \rho g \int B \xi d\xi$$

where the integrations are taken over the length of the hull.

c and G , the coefficients of the displacement in heave z in the

force and moment equations respectively, depend only on the changes in the displacement of the ship. With the linearizing assumption these are evaluated on the basis of the beam of a section. g and C , the coefficients of the angular displacement θ , depend on displacement changes and also on the kinematics of the fluid flow resulting from the ship being at an instantaneous angle of trim.

It is seen that the damping force coefficient b in heave is a function of frequency of encounter ω_e but is independent of forward speed V per se. This appears to be confirmed by the experimental work of Golovato (1956). However, the damping moment coefficient B in pitch and the cross-coupling coefficients e and E are composed of dynamic terms proportional to V and dissipative terms independent of V (except as the frequency of encounter ω_e is a function of V). While the dynamic terms contribute only a little to B , they make very important contributions to the cross-coupling coefficients.

TABLE 1
MODEL PROPERTIES

	Model Numbers							
	1445	1616	1444	1723	1699A	1699B	1699C	1699D
Length BP, ft.	5.00	5.00	4.79	5.71	4.17	-	5.55	-
Load WL, ft.	-	-	-	-	4.11	3.94	-	-
Rated LWL*, ft.	-	-	-	-	4.28	4.28	5.71	5.71
Beam, ft.	0.667	0.667	0.65	0.608	0.74	1.22	0.555	0.709
Draft, ft.	0.267	0.267	0.290	0.208	0.330	0.690	0.255	0.417
Displacement(FW), lb.	33.3	33.3	41.0	24.5	33.3	50.8	24.5	24.5
Block Coefficient	0.60	0.60	0.74	0.55	0.51	0.23	0.51	0.23
$\Delta/(\text{.01L})^3$	122	122	171	60	204	370	60	60
Radius of Gyration in air, ft.	1.27	1.27	1.15	1.37	1.07	1.07	1.37	1.37
Natural Pitching Period, afloat in calm water, sec.	0.72	0.65	0.70	0.59	0.74	0.81	0.59	0.63
Natural Heaving Period, afloat in calm water, sec.	0.70	0.70	0.75	0.61	0.73	0.78	0.61	0.60

*By Cruising Club Rule: Rated LWL = $0.3 \text{ LWL} + 0.7 (\text{LWL}_{1.04})$, where LWL is the load WL length and $\text{LWL}_{1.04}$ is the length on the WL at a draft of $1.04 \times$ load draft.

TABLE 2

VALUES OF COEFFICIENTS AND EXCITING FORCES OF
EQUATIONS OF MOTION IN HEAVING AND PITCHING
FOR ETT MODEL NO. 1145 in WAVE LENGTH = SHIP LENGTH

Symbols	Values of parameters obtained at model speed (in fps) of				
	0	1.25	2.25	3.25	4.00
ω	6.36	7.93	9.19	10.45	11.42
a	1.87	1.76	1.74	1.76	1.77
A	2.40	2.28	2.23	2.22	2.22
b	5.6	4.7	3.9	3.0	2.3
B	7.7	6.8	6.2	5.2	4.6
c	← 147 →				
C_0	← 186 →				
$C_1 + C_2$	0	2	1	-1	-5
C	186	188	187	185	181
d = D	-0.05	-0.05	-0.05	-0.05	-0.06
$e_2 = E_2$	-2.3	-2.2	-2.1	-2.0	-1.3
$e_1 = -E_1$	0	-0.9	-1.6	-2.4	-3.0
e	-2.3	-3.1	-3.7	-4.4	-4.8
E	-2.3	-1.3	-0.5	0.4	1.2
g_0	← -17 →				
g_2	0	-6	-9	-10	-9
g	-17	-23	-26	-27	-26
G	← -17 →				
$\#H$	1.0	1.0	1.0	0.8	0.8
$\#H$	3.6i	3.7i	3.7i	3.6i	3.5i

*Based on $\omega t = 0$ when wave crest is at station 11. The correction of 13° is to be applied to phase lags to convert to $\omega t = 0$ at station 10.

TABLE 3

VALUES OF COEFFICIENTS AND EXCITING FORCES OF
EQUATIONS OF MOTION IN HEAVING AND PITCHING
FOR BTT MODEL NO. 1616 IN WAVE LENGTH = SHIP LENGTH

Values of parameters obtained at model speed (in fps) of

Symbols	0	1.25	2.25	3.25	4.00
ω	6.36	7.93	9.19	10.45	11.42
a	1.91	1.77	1.75	1.77	1.79
A	2.52	2.33	2.33	2.33	2.33
b	7.1	5.9	4.9	4.1	3.4
B	11.1	10.6	9.7	8.6	7.7
c	← 160 →				
C_0	← 227 →				
$C_1 + C_2$	0	-1	-1	-3	-12
C	227	226	223	219	215
$d = D$.015	.015	.015	.014	.013
$e_2 = E_2$	-.1	0	0	0	0
$e_1 = -E_1$	0	-.9	-1.6	-2.4	-3.0
e	-.1	-.9	-1.6	-2.4	-3.0
E	-.1	.9	1.6	2.4	3.0
E_0	← 5.2 →				
E_2	0	-7.4	-11.1	-13.3	-13.6
E	5.2	-2.2	-5.9	-6.1	-6.4
G	← 5.2 →				
H	0.3	0.3	0.7	0.6	0.5
M	4.11	4.11	4.11	4.01	3.91

TABLE 4

COMPARISON OF MEAN-MAXIMUM* DESTROYER MODEL MOTIONS
IN TOWING TANK IRREGULAR HEAD SEAS,
AS OBTAINED BY THREE DIFFERENT METHODS

	Heave, inches		Pitch, degrees	
	0 speed	2.53 ft/sec.	0 speed	2.53 ft/sec
By direct analysis of model motions in irregular waves	1.1	1.3	5.2	6.7
By calculation from wave spectra and experimentally measured responses to regular waves	1.1	1.2	4.3	6.1
By calculation from wave spectra and analytically computed responses to regular waves	1.1	1.3	5.0	5.8

* Mean of 1/10 highest amplitudes

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TABLE 5

VALUES OF $\omega^2 L/g$ FOR SYNCHRONOUS SPEED, L/B , k_2 and k_1 OF THE VARIOUS
SHIP FORMS FOR COMPARISON WITH HATLOCK'S SUBMERGED SPEEDS

Model	L/B	k_2^*	k_1^*	ω		$\omega^2 L/g$	
				Pitch	Heave	Pitch	Heave
Havelock's Submerged Spheroid	8	.985	.82	-	-	-	-
Series 50 (Model 1145)	7.5	.72	.62	3.7	9.0	11.7	12.5
T-box (Model 1616)	7.5	.72	.63	9.7	9.0	11.5	12.5
T-2 Tanker (Model 1144)	7.2	.68	.51	9.0	3.2	12.0	10.2
Destroyer (Model 1723)	9.4	.90	.75	10.6	10.3	20.1	18.3
Trawler (Model 1699A)	5.3	.66	.51	3.2	3.6	9.5	9.9
Yacht (Model 1699B)	3.5	.70	.45	7.7	3.1	7.9	3.6
Lengthened Trawler (Model 1699C)	10.3	.64	.53	10.6	10.3	20.1	18.3
Lengthened Yacht (Model 1699D)	3.0	.72	.43	10.0	10.5	17.6	19.4

* k_2 for the ship forms (with free surface correction k_2) = $\frac{\text{Added Mass}}{\text{Mass of Water Displaced by a ship}}$

$k_1 = \frac{\text{Added Moment of Inertia}}{\text{Moment of Inertia of Water Displaced by a Ship.}}$

E R R A T A

Figs. 6 and 7: Substitute $-e_2 = -R_2$ for
 $-e_1 = -R_1$ (corrected for
Transactions of SNAME).

Fig. 8 : Observed data on phase relation-
ships omitted for Model 1616,
 $\lambda/L = 1.50$, will be shown in
SNAME Transactions.

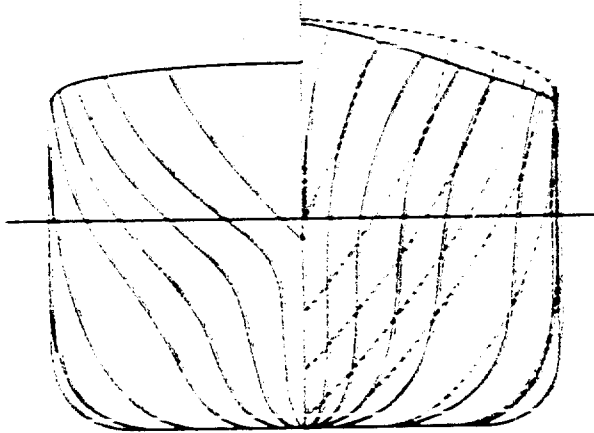


Fig. 1 — Model 445 (Series 60)
— Model 614 (V-Bow)

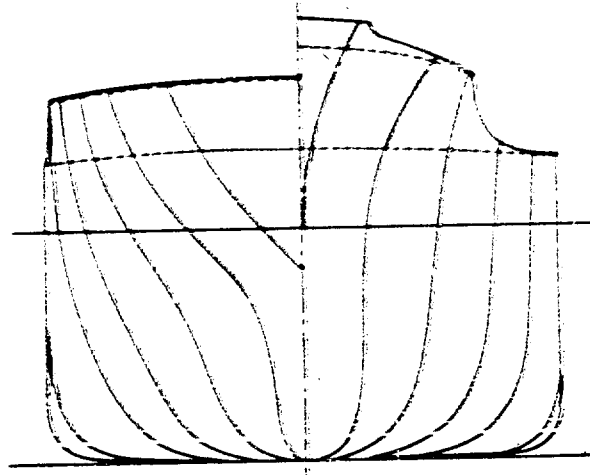


Fig. 2 Model 444 (T-2 Torpedo)

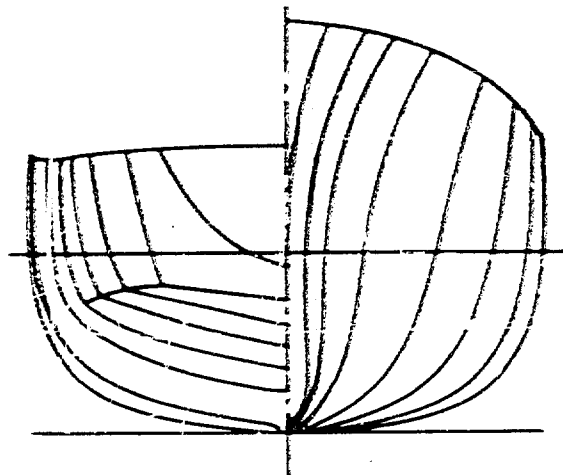


Fig. 3 Model 723 (Destroyer)

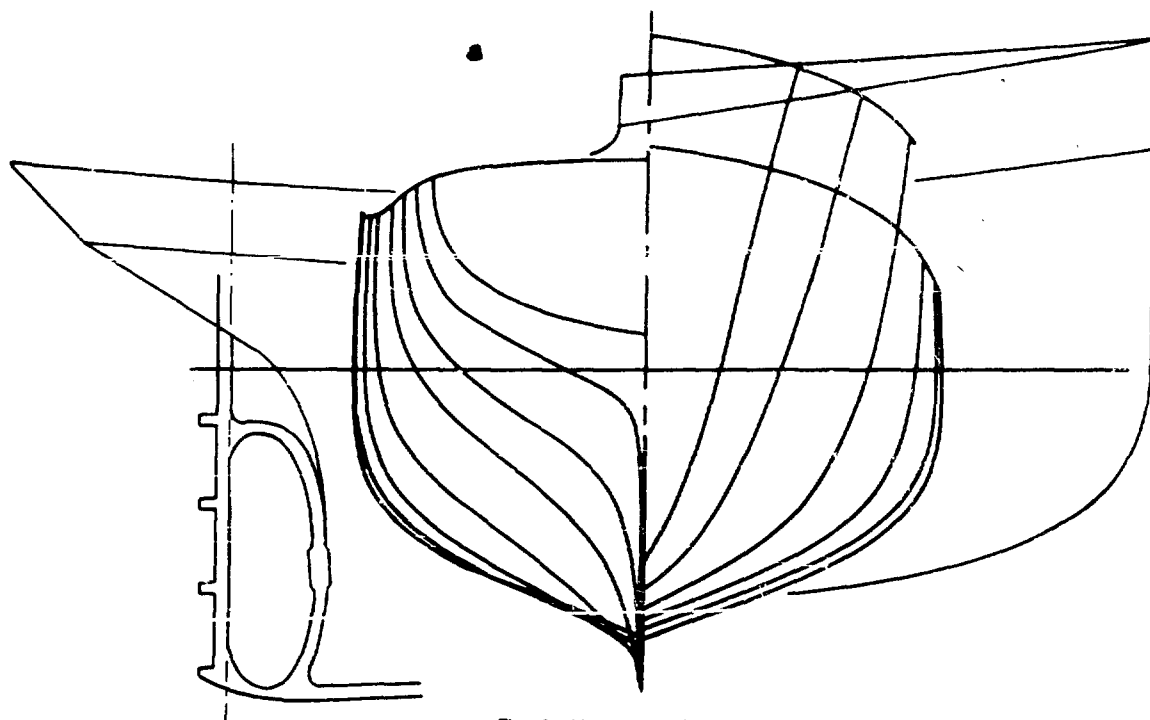


Fig. 4 Model 1699A (Trawler)

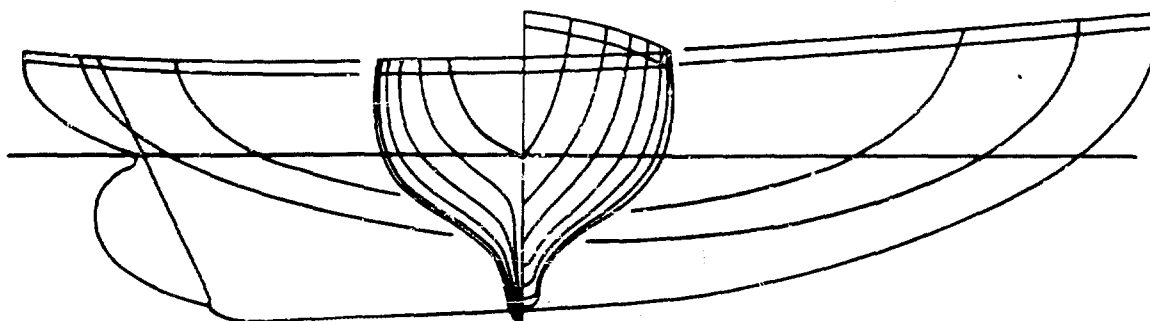


Fig. 5 Model 1699B (Yacht)

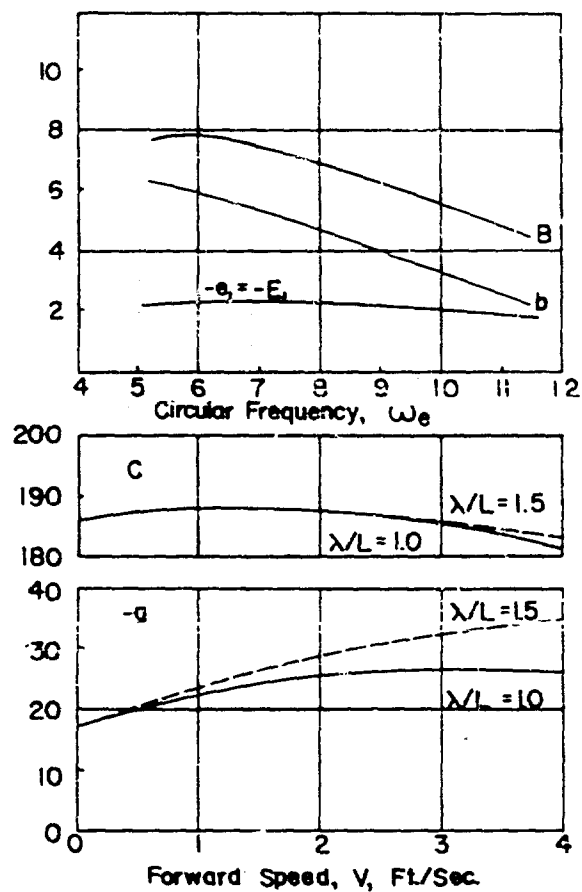


FIG. 6 Computed Coefficients
for E.T.T. Model 1445

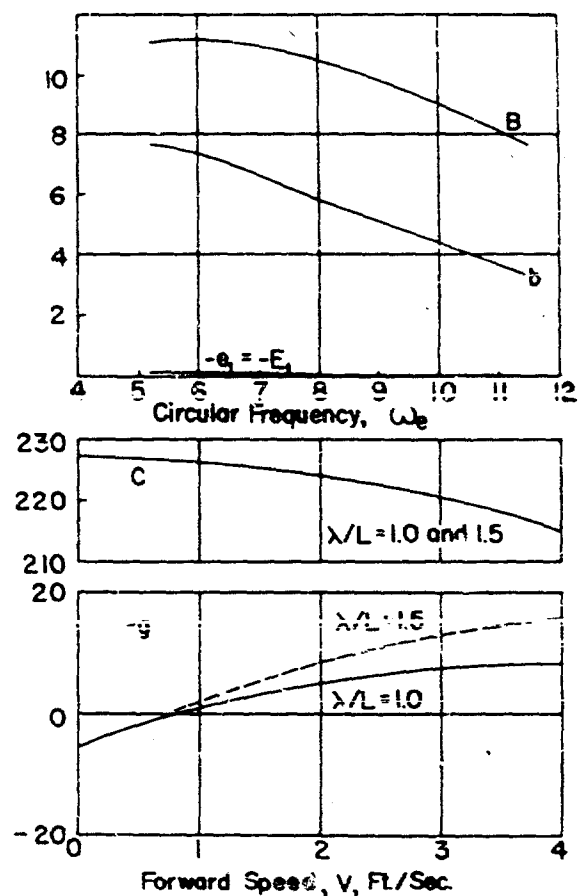


FIG. 7 Computed Coefficients
for E.T.T. Model 1616

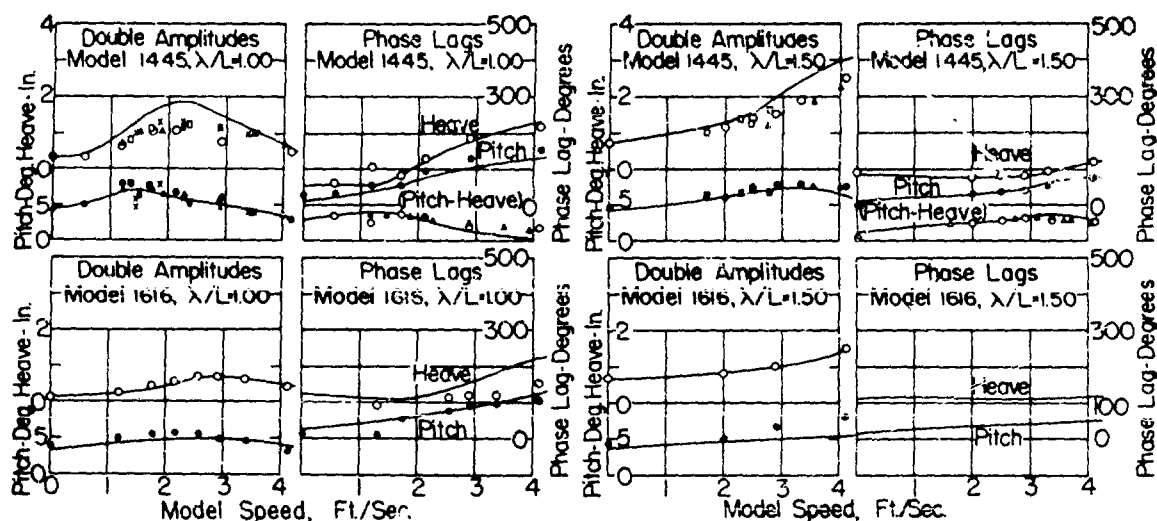


FIG. 8 Motions of 5-Ft. E.T.T. Models 1445 and 1616 in Waves of 1.0 and 1.5 Model Lengths by 1.25 in. High.*

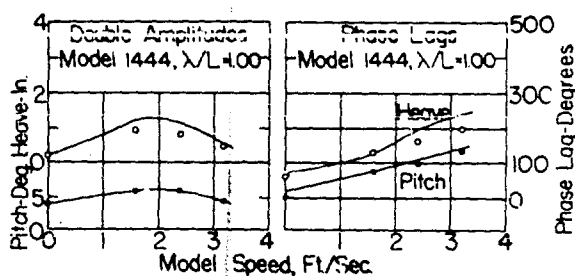


FIG. 9 Motions of 4.8-Ft. E.T.T. Model 1444 in Waves 4.8 ft. Long by 1.2 in. High.*

* Circles indicate E.T.T. experimental data (open for heave, solid for pitch) and curves show calculated motions. Zero phase lag corresponds to maximum pitch or heave with wave nodal point at midsection. (In Fig. 8 triangles are D.T.M.B. data, squares M.I.T. and exes U.of Cal.)

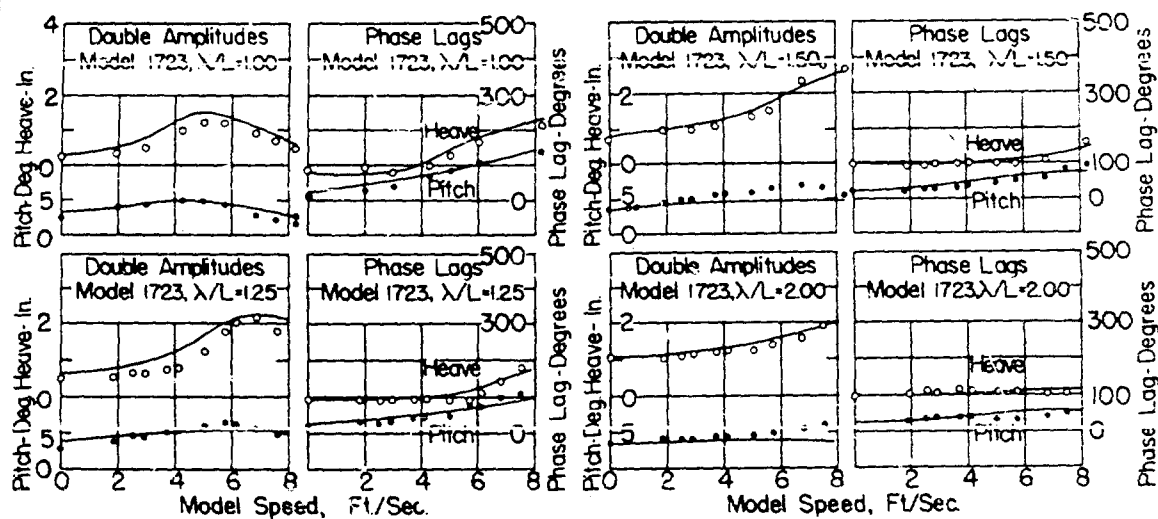


FIG. 10 Motions of 5.7-Ft. E.T.T. Model 1723 in waves of 1.0, 1.25, 1.5, and 2.0 Model Lengths by 1.43 in. High.*

*Circles indicate E.T.T. experimental data (open for heave, solid for pitch) and curves show calculated motions. Zero phase lag corresponds to maximum pitch or heave with wave nodal point at midsection.

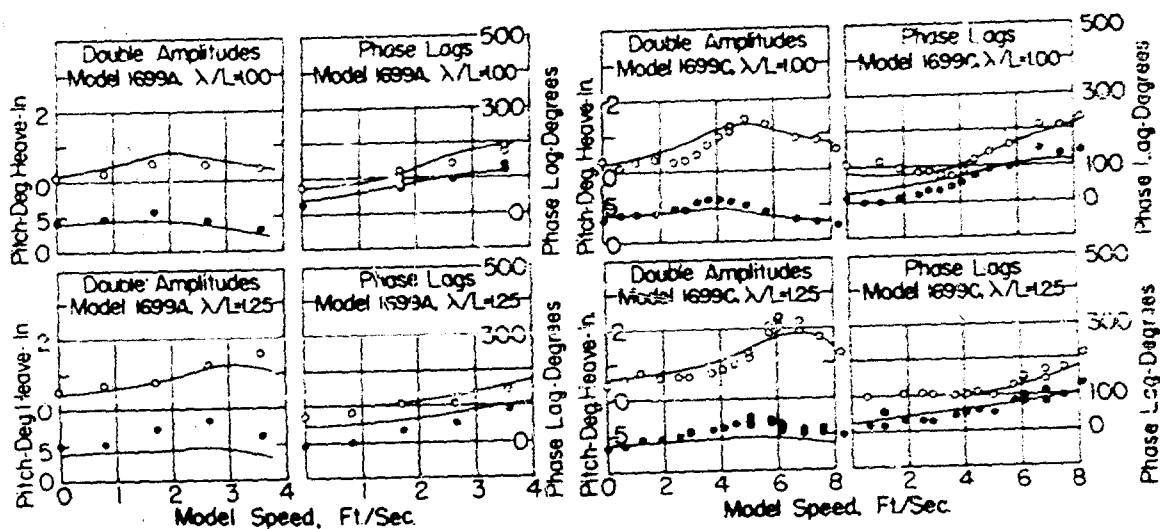


FIG. 11 Motions of 4.3-Ft. E.T.T. Model 1699A and 5.7-Ft. E.T.T. Model 1699C in Waves of 1.0 and 1.25 Model Lengths, $1/48$ Model Length in Height.*

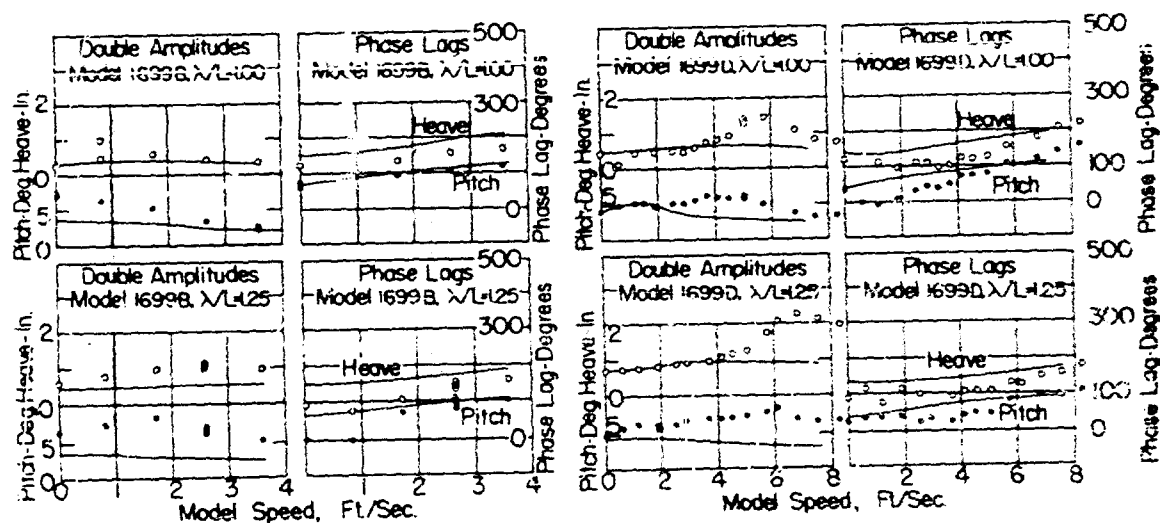


FIG. 12 Motions of 4.3-Ft. E.T.T. Model 1699B and 5.7-Ft. E.T.T. Model 1699D in Waves of 1.0 and 1.25 Model Lengths, $1/48$ Model Length in Height.*

*For note on symbols see preceding page.

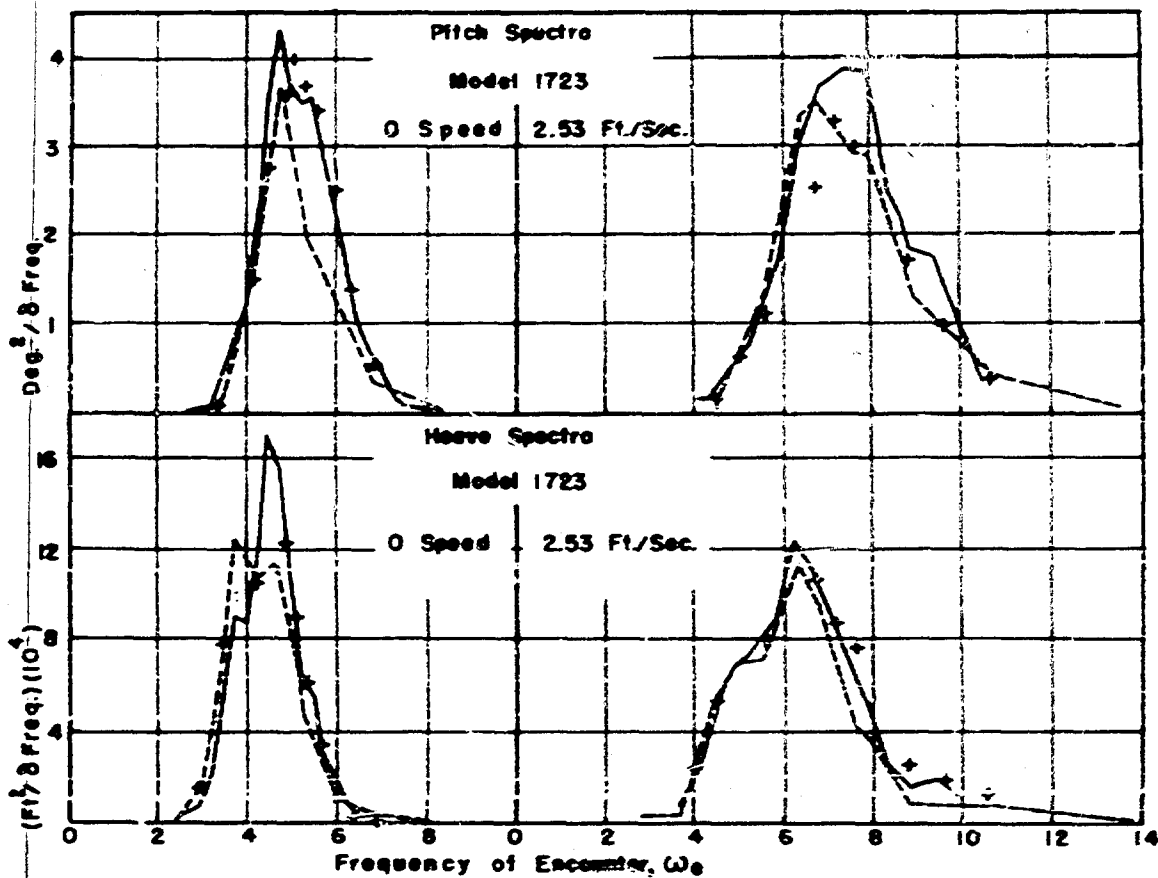


FIG. 13 Motion Spectra Predicted by Three Different Methods for Destroyer Model 1723.

- 1) By direct analysis of motions in irregular waves (solid lines).
- 2) By calculation from wave spectra and experimentally measured responses to regular waves (dash lines).
- 3) By calculation from wave spectra and analytically computed responses to regular waves (+).

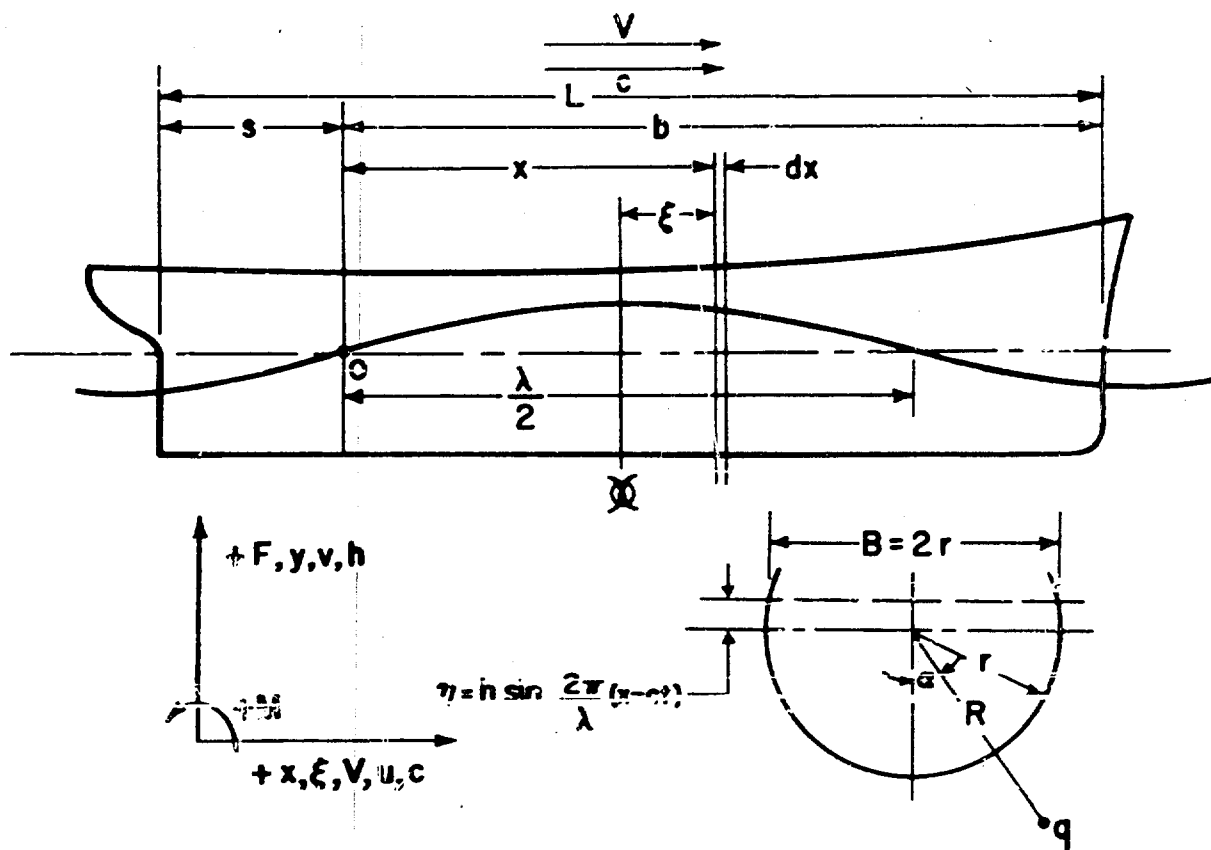


FIG. 14 Sketch Illustrating Notation Used in Appendix.

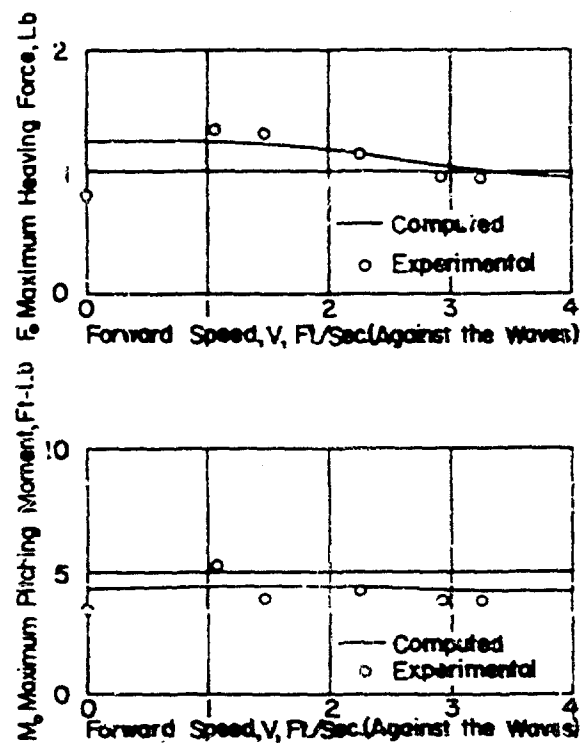


FIG. 15 Comparison of Computed and Experimentally Measured Exciting Force and Moment Amplitudes for E.T.T. Model 1145 in Waves 5 Ft. by 1.5 in.

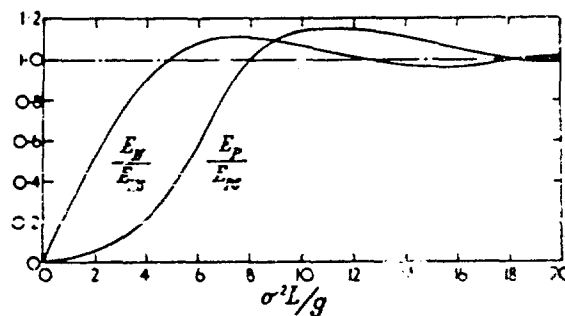


FIG. 16 Ratios of Three-Dimensional to Two-Dimensional Calculations of Damping Coefficients for Submerged Spheroid of $L/B = 8$. (Fig. 1 of Havelock, 1956)

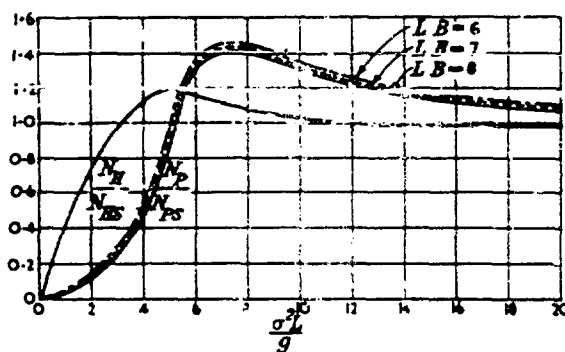


FIG. 2

FIG. 17 Ratios of Three-Dimensional to Two-Dimensional Calculations of Damping Coefficients for Thin "Michell" Ship of $B/H = 2$. (Fig. 2 of Vossers, 1956)